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MATLAB has a wide variety of functions useful to the genetic algorithm practitioner and those wishing to experiment with the genetic algorithm for the first time. Given the versatility of MATLAB’s high-level language, problems can be coded in m-files in a fraction of the time that it would take to create C or Fortran programs for the same purpose. Couple this with MATLAB’s advanced data analysis, visualisation tools and special purpose application domain toolboxes and the user is presented with a uniform environment with which to explore the potential of genetic algorithms.

The Genetic Algorithm Toolbox uses MATLAB matrix functions to build a set of versatile tools for implementing a wide range of genetic algorithm methods. The Genetic Algorithm Toolbox is a collection of routines, written mostly in m-files, which implement the most important functions in genetic algorithms.
Installation

Instructions for installing the Genetic Algorithm Toolbox can be found in the MATLAB installation instructions. It is recommended that the files for this toolbox are stored in a directory named genetic off the main matlab/toolbox directory.

A number of demonstrations are available. A single-population binary-coded genetic algorithm to solve a numerical optimization problem is implemented in the m-file sga.m. The demonstration m-file mpga.m implements a real-valued multi-population genetic algorithm to solve a dynamic control problem. Both of these demonstration m-files are discussed in detail in the Examples Section.

Additionally, a set of test functions, drawn from the genetic algorithm literature, are supplied in a separate directory, test_fns, from the Genetic Algorithm Toolbox functions. A brief description of these test functions is given at the end of the Examples Section. A further document describes the implementation and use of these functions.
An Overview of Genetic Algorithms

In this Section we give a tutorial introduction to the basic Genetic Algorithm (GA) and outline the procedures for solving problems using the GA.

What are Genetic Algorithms?

The GA is a stochastic global search method that mimics the metaphor of natural biological evolution. GAs operate on a population of potential solutions applying the principle of survival of the fittest to produce (hopefully) better and better approximations to a solution. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation.

Individuals, or current approximations, are encoded as strings, *chromosomes*, composed over some alphabet(s), so that the *genotypes* (chromosome values) are uniquely mapped onto the decision variable (*phenotypic*) domain. The most commonly used representation in GAs is the binary alphabet \{0, 1\} although other representations can be used, e.g. ternary, integer, real-valued etc. For example, a problem with two variables, \(x_1\) and \(x_2\), may be mapped onto the chromosome structure in the following way:

```
  1 0 1 1 0 1 0 0 1 1 | 0 1 0 1 1 1 0 1 0 1 0 0 1 0 1
```

where \(x_1\) is encoded with 10 bits and \(x_2\) with 15 bits, possibly reflecting the level of accuracy or range of the individual decision variables. Examining the chromosome string in isolation yields no information about the problem we are trying to solve. It is only with the decoding of the chromosome into its phenotypic values that any meaning can be applied to the representation. However, as described below, the search process will operate on this encoding of the decision variables, rather than the decision variables themselves, except, of course, where real-valued genes are used.

Having decoded the chromosome representation into the decision variable domain, it is possible to assess the performance, or *fitness*, of individual members of a population. This is done through an objective function that characterises an individual’s performance in the problem domain. In the natural world, this would be an individual’s ability to survive in its present environment. Thus, the objective
function establishes the basis for selection of pairs of individuals that will be mated together during reproduction.

During the reproduction phase, each individual is assigned a fitness value derived from its raw performance measure given by the objective function. This value is used in the selection to bias towards more fit individuals. Highly fit individuals, relative to the whole population, have a high probability of being selected for mating whereas less fit individuals have a correspondingly low probability of being selected.

Once the individuals have been assigned a fitness value, they can be chosen from the population, with a probability according to their relative fitness, and recombined to produce the next generation. Genetic operators manipulate the characters (genes) of the chromosomes directly, using the assumption that certain individual’s gene codes, on average, produce fitter individuals. The recombination operator is used to exchange genetic information between pairs, or larger groups, of individuals. The simplest recombination operator is that of single-point crossover.

Consider the two parent binary strings:

\[ P_1 = 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0, \text{ and} \]
\[ P_2 = 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0. \]

If an integer position, \( i \), is selected uniformly at random between 1 and the string length, \( l \), minus one [1, \( l-1 \)], and the genetic information exchanged between the individuals about this point, then two new offspring strings are produced. The two offspring below are produced when the crossover point \( i = 5 \) is selected,

\[ O_1 = 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0, \text{ and} \]
\[ O_2 = 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0. \]

This crossover operation is not necessarily performed on all strings in the population. Instead, it is applied with a probability \( P_x \) when the pairs are chosen for breeding. A further genetic operator, called mutation, is then applied to the new chromosomes, again with a set probability, \( P_m \). Mutation causes the individual genetic representation to be changed according to some probabilistic rule. In the binary string representation, mutation will cause a single bit to change its state, \( 0 \rightarrow 1 \) or \( 1 \rightarrow 0 \). So, for example, mutating the fourth bit of \( O_1 \) leads to the new string,

\[ O_{1m} = 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0. \]

Mutation is generally considered to be a background operator that ensures that the probability of searching a particular subspace of the problem space is never zero.
This has the effect of tending to inhibit the possibility of converging to a local optimum, rather than the global optimum.

After recombination and mutation, the individual strings are then, if necessary, decoded, the objective function evaluated, a fitness value assigned to each individual and individuals selected for mating according to their fitness, and so the process continues through subsequent generations. In this way, the average performance of individuals in a population is expected to increase, as good individuals are preserved and bred with one another and the less fit individuals die out. The GA is terminated when some criteria are satisfied, e.g. a certain number of generations, a mean deviation in the population, or when a particular point in the search space is encountered.

**GAs versus Traditional Methods**

From the above discussion, it can be seen that the GA differs substantially from more traditional search and optimization methods. The four most significant differences are:

- GAs search a population of points in parallel, not a single point.
- GAs do not require derivative information or other auxiliary knowledge; only the objective function and corresponding fitness levels influence the directions of search.
- GAs use probabilistic transition rules, not deterministic ones.
- GAs work on an encoding of the parameter set rather than the parameter set itself (except in where real-valued individuals are used).

It is important to note that the GA provides a number of potential solutions to a given problem and the choice of final solution is left to the user. In cases where a particular problem does not have one individual solution, for example a family of Pareto-optimal solutions, as is the case in multiobjective optimization and scheduling problems, then the GA is potentially useful for identifying these alternative solutions simultaneously.
Major Elements of the Genetic Algorithm

The simple genetic algorithm (SGA) is described by Goldberg [1] and is used here to illustrate the basic components of the GA. A pseudo-code outline of the SGA is shown in Fig. 1. The population at time \( t \) is represented by the time-dependent variable \( P_t \), with the initial population of random estimates being \( P_t(0) \). Using this outline of a GA, the remainder of this Section describes the major elements of the GA.

```
procedure GA
begin
  t = 0;
  initialize P(t);
  evaluate P(t);
  while not finished do
    begin
      t = t + 1;
      select P(t) from P(t-1);
      reproduce pairs in P(t);
      evaluate P(t);
    end
  end
end.
```

*Figure 1: A Simple Genetic Algorithm*

Population Representation and Initialisation

GAs operate on a number of potential solutions, called a population, consisting of some encoding of the parameter set simultaneously. Typically, a population is composed of between 30 and 100 individuals, although, a variant called the micro GA uses very small populations, ~10 individuals, with a restrictive reproduction and replacement strategy in an attempt to reach real-time execution [2].

The most commonly used representation of chromosomes in the GA is that of the single-level binary string. Here, each decision variable in the parameter set is encoded as a binary string and these are concatenated to form a chromosome. The use of Gray coding has been advocated as a method of overcoming the hidden representational bias in conventional binary representation as the Hamming distance between adjacent values is constant [3]. Empirical evidence of Caruana and Schaffer [4] suggests that large Hamming distances in the representational mapping between adjacent values, as is the case in the standard binary representation, can result in the search process being deceived or unable to
efficiently locate the global minimum. A further approach of Schmitendorf et-al [5], is the use of logarithmic scaling in the conversion of binary-coded chromosomes to their real phenotypic values. Although the precision of the parameter values is possibly less consistent over the desired range, in problems where the spread of feasible parameters is unknown, a larger search space may be covered with the same number of bits than a linear mapping scheme allowing the computational burden of exploring unknown search spaces to be reduced to a more manageable level.

Whilst binary-coded GAs are most commonly used, there is an increasing interest in alternative encoding strategies, such as integer and real-valued representations. For some problem domains, it is argued that the binary representation is in fact deceptive in that it obscures the nature of the search [6]. In the subset selection problem [7], for example, the use of an integer representation and look-up tables provides a convenient and natural way of expressing the mapping from representation to problem domain.

The use of real-valued genes in GAs is claimed by Wright [8] to offer a number of advantages in numerical function optimization over binary encodings. Efficiency of the GA is increased as there is no need to convert chromosomes to phenotypes before each function evaluation; less memory is required as efficient floating-point internal computer representations can be used directly; there is no loss in precision by discretisation to binary or other values; and there is greater freedom to use different genetic operators. The use of real-valued encodings is described in detail by Michalewicz [9] and in the literature on Evolution Strategies (see, for example, [10]).

Having decided on the representation, the first step in the SGA is to create an initial population. This is usually achieved by generating the required number of individuals using a random number generator that uniformly distributes numbers in the desired range. For example, with a binary population of $N_{ind}$ individuals whose chromosomes are $L_{ind}$ bits long, $N_{ind} \times L_{ind}$ random numbers uniformly distributed from the set $\{0, 1\}$ would be produced.

A variation is the extended random initialisation procedure of Bramlette [6] whereby a number of random initialisations are tried for each individual and the one with the best performance is chosen for the initial population. Other users of GAs have seeded the initial population with some individuals that are known to be in the vicinity of the global minimum (see, for example, [11] and [12]). This approach is, of course, only applicable if the nature of the problem is well understood beforehand or if the GA is used in conjunction with a knowledge based system.

The GA Toolbox supports binary, integer and floating-point chromosome representations. Binary and integer populations may be initialised using the Toolbox function to create binary populations, crtpb. An additional function, crtbase, is provided that builds a vector describing the integer representation
used. Real-valued populations may be initialised with the function `crtp`. Conversion between binary strings and real values is provided by the routine `bs2rv` that supports the use of Gray codes and logarithmic scaling.

**The Objective and Fitness Functions**

The objective function is used to provide a measure of how individuals have performed in the problem domain. In the case of a minimization problem, the most fit individuals will have the lowest numerical value of the associated objective function. This raw measure of fitness is usually only used as an intermediate stage in determining the relative performance of individuals in a GA. Another function, the *fitness function*, is normally used to transform the objective function value into a measure of relative fitness [13], thus:

\[ F(x) = g(f(x)) \]

where \( f \) is the objective function, \( g \) transforms the value of the objective function to a non-negative number and \( F \) is the resulting relative fitness. This mapping is always necessary when the objective function is to be minimized as the lower objective function values correspond to fitter individuals. In many cases, the fitness function value corresponds to the number of offspring that an individual can expect to produce in the next generation. A commonly used transformation is that of proportional fitness assignment (see, for example, [1]). The individual fitness, \( F(x_i) \), of each individual is computed as the individual’s raw performance, \( f(x_i) \), relative to the whole population, i.e.,

\[ F(x_i) = \frac{f(x_i)}{N_{ind} \sum_{i=1}^{N_{ind}} f(x_i)} \]

where \( N_{ind} \) is the population size and \( x_i \) is the phenotypic value of individual \( i \). Whilst this fitness assignment ensures that each individual has a probability of reproducing according to its relative fitness, it fails to account for negative objective function values.

A linear transformation which offsets the objective function [1] is often used prior to fitness assignment, such that,

\[ F(x) = af(x) + b \]

where \( a \) is a positive scaling factor if the optimization is maximizing and negative if we are minimizing. The offset \( b \) is used to ensure that the resulting fitness values are non-negative.
The linear scaling and offsetting outlined above is, however, susceptible to rapid convergence. The selection algorithm (see below) selects individuals for reproduction on the basis of their relative fitness. Using linear scaling, the expected number of offspring is approximately proportional to that individuals performance. As there is no constraint on an individual’s performance in a given generation, highly fit individuals in early generations can dominate the reproduction causing rapid convergence to possibly sub-optimal solutions. Similarly, if there is little deviation in the population, then scaling provides only a small bias towards the most fit individuals.

Baker [14] suggests that by limiting the reproductive range, so that no individuals generate an excessive number of offspring, prevents premature convergence. Here, individuals are assigned a fitness according to their rank in the population rather than their raw performance. One variable, $MAX$, is used to determine the bias, or selective pressure, towards the most fit individuals and the fitness of the others is determined by the following rules:

- $MIN = 2.0 - MAX$
- $INC = 2.0 \times (MAX - 1.0) / N_{ind}$
- $LOW = INC / 2.0$

where $MIN$ is the lower bound, $INC$ is the difference between the fitness of adjacent individuals and $LOW$ is the expected number of trials (number of times selected) of the least fit individual. $MAX$ is typically chosen in the interval [1.1, 2.0]. Hence, for a population size of $N_{ind} = 40$ and $MAX = 1.1$, we obtain $MIN = 0.9$, $INC = 0.05$ and $LOW = 0.025$. The fitness of individuals in the population may also be calculated directly as,

$$F(x_i) = 2 - MAX + 2 \times (MAX - 1) \times \frac{x_i - 1}{N_{ind} - 1},$$

where $x_i$ is the position in the ordered population of individual $i$.

Objective functions must be created by the user, although a number of example m-files are supplied with the Toolbox that implement common test functions. These objective functions all have the filename prefix obj. The Toolbox supports both linear and non-linear ranking methods, ranking, and includes a simple linear scaling function, scaling, for completeness. It should be noted that the linear scaling function is not suitable for use with objective functions that return negative fitness values.

**Selection**

Selection is the process of determining the number of times, or trials, a particular individual is chosen for reproduction and, thus, the number of offspring that an
individual will produce. The selection of individuals can be viewed as two separate processes:

1) determination of the number of trials an individual can expect to receive, and
2) conversion of the expected number of trials into a discrete number of offspring.

The first part is concerned with the transformation of raw fitness values into a real-valued expectation of an individual’s probability to reproduce and is dealt with in the previous subsection as fitness assignment. The second part is the probabilistic selection of individuals for reproduction based on the fitness of individuals relative to one another and is sometimes known as sampling. The remainder of this subsection will review some of the more popular selection methods in current usage.

Baker [15] presented three measures of performance for selection algorithms, bias, spread and efficiency. Bias is defined as the absolute difference between an individual’s actual and expected selection probability. Optimal zero bias is therefore achieved when an individual’s selection probability equals its expected number of trials.

Spread is the range in the possible number of trials that an individual may achieve. If \( f(i) \) is the actual number of trials that individual \( i \) receives, then the “minimum spread” is the smallest spread that theoretically permits zero bias, i.e.

\[
f(i) \in \left\{ \lfloor et(i) \rfloor, \lceil et(i) \rceil \right\}
\]

where \( et(i) \) is the expected number of trials of individual \( i \). \( \lfloor et(i) \rfloor \) is the floor of \( et(i) \) and \( \lceil et(i) \rceil \) is the ceil. Thus, while bias is an indication of accuracy, the spread of a selection method measures its consistency.

The desire for efficient selection methods is motivated by the need to maintain a GAs overall time complexity. It has been shown in the literature that the other phases of a GA (excluding the actual objective function evaluations) are \( O(L_{ind}N_{ind}) \) or better time complexity, where \( L_{ind} \) is the length of an individual and \( N_{ind} \) is the population size. The selection algorithm should thus achieve zero bias whilst maintaining a minimum spread and not contributing to an increased time complexity of the GA.

**Roulette Wheel Selection Methods**

Many selection techniques employ a “roulette wheel” mechanism to probabilistically select individuals based on some measure of their performance. A real-valued interval, \( Sum \), is determined as either the sum of the individuals’
expected selection probabilities or the sum of the raw fitness values over all the individuals in the current population. Individuals are then mapped one-to-one into contiguous intervals in the range [0, \( Sum \)]. The size of each individual interval corresponds to the fitness value of the associated individual. For example, in Fig. 2 the circumference of the roulette wheel is the sum of all six individual’s fitness values. Individual 5 is the most fit individual and occupies the largest interval, whereas individuals 6 and 4 are the least fit and have correspondingly smaller intervals within the roulette wheel. To select an individual, a random number is generated in the interval [0, \( Sum \)] and the individual whose segment spans the random number is selected. This process is repeated until the desired number of individuals have been selected.

The basic roulette wheel selection method is stochastic sampling with replacement (SSR). Here, the segment size and selection probability remain the same throughout the selection phase and individuals are selected according to the procedure outlined above. SSR gives zero bias but a potentially unlimited spread. Any individual with a segment size > 0 could entirely fill the next population.

![Figure 2: Roulette Wheel Selection](image)

Stochastic sampling with partial replacement (SSPR) extends upon SSR by resizing an individual’s segment if it is selected. Each time an individual is selected, the size of its segment is reduced by 1.0. If the segment size becomes negative, then it is set to 0.0. This provides an upper bound on the spread of \( \lfloor et(i) \rfloor \). However, the lower bound is zero and the bias is higher than that of SSR.

Remainder sampling methods involve two distinct phases. In the integral phase, individuals are selected deterministically according to the integer part of their expected trials. The remaining individuals are then selected probabilistically from the fractional part of the individuals expected values. Remainder stochastic sampling with replacement (RSSR) uses roulette wheel selection to sample the individual not assigned deterministically. During the roulette wheel selection phase, individual’s fractional parts remain unchanged and, thus, compete for
selection between “spins”. RSSR provides zero bias and the spread is lower bounded. The upper bound is limited only by the number of fractionally assigned samples and the size of the integral part of an individual. For example, any individual with a fractional part > 0 could win all the samples during the fractional phase. Remainder stochastic sampling without replacement (RSSWR) sets the fractional part of an individual’s expected values to zero if it is sampled during the fractional phase. This gives RSSWR minimum spread, although this selection method is biased in favour of smaller fractions.

**Stochastic Universal Sampling**

Stochastic universal sampling (SUS) is a single-phase sampling algorithm with minimum spread and zero bias. Instead of the single selection pointer employed in roulette wheel methods, SUS uses \( N \) equally spaced pointers, where \( N \) is the number of selections required. The population is shuffled randomly and a single random number in the range \([0 \sum i/N]\) is generated, \( ptr \). The \( N \) individuals are then chosen by generating the \( N \) pointers spaced by 1, \([ptr, ptr+1, ..., ptr+N-1]\), and selecting the individuals whose fitnesses span the positions of the pointers. An individual is thus guaranteed to be selected a minimum of \( \lceil et(i) \rceil \) times and no more than \( \lfloor et(i) \rfloor \), thus achieving minimum spread. In addition, as individuals are selected entirely on their position in the population, SUS has zero bias.

The roulette wheel selection methods can all be implemented as \( O(N\log N) \) although SUS is a simpler algorithm and has time complexity \( O(N) \). The Toolbox supplies a stochastic universal sampling function, \texttt{sus}, and the stochastic sampling with replacement algorithm, \texttt{rws}.

**Crossover (Recombination)**

The basic operator for producing new chromosomes in the GA is that of crossover. Like its counterpart in nature, crossover produces new individuals that have some parts of both parent’s genetic material. The simplest form of crossover is that of single-point crossover, described in the Overview of GAs. In this Section, a number of variations on crossover are described and discussed and the relative merits of each reviewed.

**Multi-point Crossover**

For multi-point crossover, \( m \) crossover positions, \( k_i \in \{1, 2, ..., l - 1\} \), where \( k_i \) are the crossover points and \( l \) is the length of the chromosome, are chosen at random with no duplicates and sorted into ascending order. Then, the bits between successive crossover points are exchanged between the two parents to produce two
new offspring. The section between the first allele position and the first crossover point is not exchanged between individuals. This process is illustrated in Fig. 3.

![Figure 3: Multi-point Crossover (m=5)](image)

The idea behind multi-point, and indeed many of the variations on the crossover operator, is that the parts of the chromosome representation that contribute to the most to the performance of a particular individual may not necessarily be contained in adjacent substrings [16]. Further, the disruptive nature of multi-point crossover appears to encourage the exploration of the search space, rather than favoring the convergence to highly fit individuals early in the search, thus making the search more robust [17].

**Uniform Crossover**

Single and multi-point crossover define cross points as places between loci where a chromosome can be split. Uniform crossover [18] generalises this scheme to make every locus a potential crossover point. A crossover mask, the same length as the chromosome structures is created at random and the parity of the bits in the mask indicates which parent will supply the offspring with which bits. Consider the following two parents, crossover mask and resulting offspring:

\[
P_1 = 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1
\]
\[
P_2 = 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0
\]
\[
\text{Mask} = 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0
\]
\[
O_1 = 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0
\]
\[
O_2 = 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1
\]

Here, the first offspring, \(O_1\), is produced by taking the bit from \(P_1\) if the corresponding mask bit is 1 or the bit from \(P_2\) if the corresponding mask bit is 0. Offspring \(O_2\) is created using the inverse of the mask or, equivalently, swapping \(P_1\) and \(P_2\).

Uniform crossover, like multi-point crossover, has been claimed to reduce the bias associated with the length of the binary representation used and the particular coding for a given parameter set. This helps to overcome the bias in single-point crossover towards short substrings without requiring precise understanding of the
significance of individual bits in the chromosome representation. Spears and De Jong [19] have demonstrated how uniform crossover may be parameterised by applying a probability to the swapping of bits. This extra parameter can be used to control the amount of disruption during recombination without introducing a bias towards the length of the representation used. When uniform crossover is used with real-valued alleles, it is usually referred to as discrete recombination.

Other Crossover Operators

A related crossover operator is that of shuffle [20]. A single cross-point is selected, but before the bits are exchanged, they are randomly shuffled in both parents. After recombination, the bits in the offspring are unshuffled. This too removes positional bias as the bits are randomly reassigned each time crossover is performed.

The reduced surrogate operator [16] constrains crossover to always produce new individuals wherever possible. Usually, this is implemented by restricting the location of crossover points such that crossover points only occur where gene values differ.

Intermediate Recombination

Given a real-valued encoding of the chromosome structure, intermediate recombination is a method of producing new phenotypes around and between the values of the parents’ phenotypes [21]. Offspring are produced according to the rule,

\[ O_1 = P_1 \times \alpha (P_2 - P_1) , \]

where \( \alpha \) is a scaling factor chosen uniformly at random over some interval, typically \([-0.25, 1.25]\) and \( P_1 \) and \( P_2 \) are the parent chromosomes (see, for example, [21]). Each variable in the offspring is the result of combining the variables in the parents according to the above expression with a new \( \alpha \) chosen for each pair of parent genes. In geometric terms, intermediate recombination is
capable of producing new variables within a slightly larger hypercube than that
defined by the parents but constrained by the range of $\alpha$, as shown in Fig 4.

![Figure 4: Geometric Effect of Intermediate Recombination](image)

**Line Recombination**

Line recombination [21] is similar to intermediate recombination, except that only
one value of $\alpha$ is used in the recombination. Fig. 5 shows how line recombination
can generate any point on the line defined by the parents within the limits of the
perturbation, $\alpha$, for a recombination in two variables.

![Figure 5: Geometric Effect of Line Recombination](image)

**Discussion**

The binary operators discussed in this Section have all, to some extent, used
disruption in the representation to help improve exploration during recombination.
Whilst these operators may be used with real-valued populations, the resulting
changes in the genetic material after recombination would not extend to the actual values of the decision variables, although offspring may, of course, contain genes from either parent. The intermediate and line recombination operators overcome this limitation by acting on the decision variables themselves. Like uniform crossover, the real-valued operators may also be parameterised to provide a control over the level of disruption introduced into offspring. For discrete-valued representations, variations on the recombination operators may be used that ensure that only valid values are produced as a result of crossover [22].

The GA Toolbox provides a number of crossover routines incorporating most of the methods described above. Single-point, double-point and shuffle crossover are implemented in the Toolbox functions xovsp, xovdp and xovsh, respectively, and can operate on any chromosome representation. Reduced surrogate crossover is supported with both single-point, xovsprs, and double-point, xovdprs, crossover and with shuffle crossover, xovshrs. A further general multi-point crossover routine, xovmp, is also provided. To support real-valued chromosome representations, discrete, intermediate and line recombination operators are also included. The discrete recombination operator, recdis, performs crossover on real-valued individuals in a similar manner to the uniform crossover operators. Line and intermediate recombination are supported by the functions reclin and recint respectively. A high-level entry function to all of the crossover operators is provided by the function recombin.

**Mutation**

In natural evolution, mutation is a random process where one allele of a gene is replaced by another to produce a new genetic structure. In GAs, mutation is randomly applied with low probability, typically in the range 0.001 and 0.01, and modifies elements in the chromosomes. Usually considered as a background operator, the role of mutation is often seen as providing a guarantee that the probability of searching any given string will never be zero and acting as a safety net to recover good genetic material that may be lost through the action of selection and crossover [1].

The effect of mutation on a binary string is illustrated in Fig. 6 for a 10-bit chromosome representing a real value decoded over the interval [0, 10] using both standard and Gray coding and a mutation point of 3 in the binary string. Here, binary mutation flips the value of the bit at the loci selected to be the mutation point. Given that mutation is generally applied uniformly to an entire population of
strings, it is possible that a given binary string may be mutated at more than one point.

<table>
<thead>
<tr>
<th></th>
<th>binary</th>
<th>Gray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original string - 0 0 1 1 0 0 0 1 0</td>
<td>0.9659</td>
<td>0.6634</td>
</tr>
<tr>
<td>Mutated string - 0 0 1 1 0 0 0 1 0</td>
<td>2.2146</td>
<td>1.8439</td>
</tr>
</tbody>
</table>

*Figure 6: Binary Mutation*

With non-binary representations, mutation is achieved by either perturbing the gene values or random selection of new values within the allowed range. Wright [8] and Janikow and Michalewicz [23] demonstrate how real-coded GAs may take advantage of higher mutation rates than binary-coded GAs, increasing the level of possible exploration of the search space without adversely affecting the convergence characteristics. Indeed, Tate and Smith [24] argue that for codings more complex than binary, high mutation rates can be both desirable and necessary and show how, for a complex combinatorial optimization problem, high mutation rates and non-binary coding yielded significantly better solutions than the normal approach.

Many variations on the mutation operator have been proposed. For example, biasing the mutation towards individuals with lower fitness values to increase the exploration in the search without losing information from the fitter individuals [25] or parameterising the mutation such that the mutation rate decreases with the population convergence [26]. Mühlenbein [21] has introduced a mutation operator for the real-coded GA that uses a non-linear term for the distribution of the range of mutation applied to gene values. It is claimed that by biasing mutation towards smaller changes in gene values, mutation can be used in conjunction with recombination as a foreground search process. Other mutation operations include that of *trade mutation* [7], whereby the contribution of individual genes in a chromosome is used to direct mutation towards weaker terms, and *reorder mutation* [7], that swaps the positions of bits or genes to increase diversity in the decision variable space.

Binary and integer mutation are provided in the Toolbox by the function `mut`. Real-valued mutation is available using the function `mutbga`. A high-level entry function to the mutation operators is provided by the function `mutate`. 
Reinsertion

Once a new population has been produced by selection and recombination of individuals from the old population, the fitness of the individuals in the new population may be determined. If fewer individuals are produced by recombination than the size of the original population, then the fractional difference between the new and old population sizes is termed a generation gap [27]. In the case where the number of new individuals produced at each generation is one or two, the GA is said to be steady-state [28] or incremental [29]. If one or more of the most fit individuals is deterministically allowed to propagate through successive generations then the GA is said to use an elitist strategy.

To maintain the size of the original population, the new individuals have to be reinserted into the old population. Similarly, if not all the new individuals are to be used at each generation or if more offspring are generated than the size of the old population then a reinsertion scheme must be used to determine which individuals are to exist in the new population. An important feature of not creating more offspring than the current population size at each generation is that the generational computational time is reduced, most dramatically in the case of the steady-state GA, and that the memory requirements are smaller as fewer new individuals need to be stored while offspring are produced.

When selecting which members of the old population should be replaced the most apparent strategy is to replace the least fit members deterministically. However, in studies, Fogarty [30] has shown that no significant difference in convergence characteristics was found when the individuals selected for replacement where chosen with inverse proportional selection or deterministically as the least fit. He further asserts that replacing the least fit members effectively implements an elitist strategy as the most fit will probabilistically survive through successive generations. Indeed, the most successful replacement scheme was one that selected the oldest members of a population for replacement. This is reported as being more in keeping with generational reproduction as every member of the population will, at some time, be replaced. Thus, for an individual to survive successive generations, it must be sufficiently fit to ensure propagation into future generations.

The GA Toolbox provides a function for reinserting individuals into the population after recombination, reins. Optional input parameters allow the use of either uniform random or fitness-based reinsertion. Additionally, this routine can also be selected to reinsert fewer offspring than those produced at recombination.

Termination of the GA

Because the GA is a stochastic search method, it is difficult to formally specify convergence criteria. As the fitness of a population may remain static for a number of generations before a superior individual is found, the application of
conventional termination criteria becomes problematic. A common practice is to terminate the GA after a prespecified number of generations and then test the quality of the best members of the population against the problem definition. If no acceptable solutions are found, the GA may be restarted or a fresh search initiated.
Data Structures

MATLAB essentially supports only one data type, a rectangular matrix of real or complex numeric elements. The main data structures in the Genetic Algorithm toolbox are:

- chromosomes
- phenotypes
- objective function values
- fitness values

These data structures are discussed in the following subsections.

Chromosomes

The chromosome data structure stores an entire population in a single matrix of size $N_{ind} \times L_{ind}$, where $N_{ind}$ is the number of individuals in the population and $L_{ind}$ is the length of the genotypic representation of those individuals. Each row corresponds to an individual’s genotype, consisting of base-$n$, typically binary, values.

An example of the chromosome data structure is shown below.

$$
\text{Chrom} = 
\begin{bmatrix}
    g_{1,1} & g_{1,2} & g_{1,3} & \cdots & g_{1,L_{ind}} \\
    g_{2,1} & g_{2,2} & g_{2,3} & \cdots & g_{2,L_{ind}} \\
    g_{3,1} & g_{3,2} & g_{3,3} & \cdots & g_{3,L_{ind}} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    g_{N_{ind},1} & g_{N_{ind},2} & g_{N_{ind},3} & \cdots & g_{N_{ind},L_{ind}}
\end{bmatrix}
$$

This data representation does not force a structure on the chromosome structure, only requiring that all chromosomes are of equal length. Thus, structured populations or populations with varying genotypic bases may be used in the Genetic Algorithm Toolbox provided that a suitable decoding function, mapping chromosomes onto phenotypes, is employed. The role of the decoding function is described below.

Phenotypes

The decision variables, or phenotypes, in the genetic algorithm are obtained by applying some mapping from the chromosome representation into the decision variable space. Here, each string contained in the chromosome structure decodes
to a row vector of order Nvar, according to the number of dimensions in the search space and corresponding to the decision variable vector value.

The decision variables are stored in a numerical matrix of size Nind x Nvar. Again, each row corresponds to a particular individual’s phenotype. An example of the phenotype data structure is given below, where DECODE is used to represent an arbitrary function, possibly from the GA Toolbox, mapping the genotypes onto the phenotypes.

\[
\text{Phen} = \text{DECODE}(\text{Chrom}) \quad \% \text{ Map Genotype to Phenotype}
\]

\[
\begin{bmatrix}
    x_{1,1} & x_{1,2} & x_{1,3} & \ldots & x_{1,N\text{var}} \\
    x_{2,1} & x_{2,2} & x_{2,3} & \ldots & x_{2,N\text{var}} \\
    x_{3,1} & x_{3,2} & x_{3,3} & \ldots & x_{3,N\text{var}} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    x_{N\text{ind},1} & x_{N\text{ind},2} & x_{N\text{ind},3} & \ldots & x_{N\text{ind},N\text{var}}
\end{bmatrix}
\]

\[
\text{individual 1} \hspace{1cm} \text{individual 2} \hspace{1cm} \text{individual 3} \hspace{1cm} \ldots \hspace{1cm} \text{individual Nind}
\]

The actual mapping between the chromosome representation and their phenotypic values depends upon the DECODE function used. It is perfectly feasible using this representation to have vectors of decision variables of different types. For example, it is possible to mix integer, real-valued and alphanumeric decision variables in the same Phen data structure.

**Objective function values**

An objective function is used to evaluate the performance of the phenotypes in the problem domain. Objective function values can be scalar or, in the case of multiobjective problems, vectorial. Note that objective function values are not necessarily the same as the fitness values.

Objective function values are stored in a numerical matrix of size Nind x Nobj, where Nobj is the number of objectives. Each row corresponds to a particular individual’s objective vector. An example of the objective function values data structure is shown below, with OBJFUN representing an arbitrary objective function.
ObjV = OBJFUN(Phen) \% Objective Function

\[
\begin{bmatrix}
  y_{1,1} & y_{1,2} & y_{1,3} & \cdots & y_{1,Nvar} \\
  y_{2,1} & y_{2,2} & y_{2,3} & \cdots & y_{2,Nvar} \\
  y_{3,1} & y_{3,2} & y_{3,3} & \cdots & y_{3,Nvar} \\
  \vdots & \vdots & \vdots & \cdots & \vdots \\
  y_{Nind,1} & y_{Nind,2} & y_{Nind,3} & \cdots & y_{Nind,Nvar}
\end{bmatrix}
\]

individual 1
individual 2
individual 3
individual Nind

**Fitness values**

Fitness values are derived from objective function values through a scaling or ranking function. Fitnesses are non-negative scalars and are stored in column vectors of length Nind, an example of which is shown below. Again, FITNESS is an arbitrary fitness function.

Fitn = FITNESS(ObjV) \% Fitness Function

\[
\begin{bmatrix}
  f_1 \\
  f_2 \\
  f_3 \\
  \vdots \\
  f_{Nind}
\end{bmatrix}
\]

individual 1
individual 2
individual 3
individual Nind

Note that for multiobjective functions, the fitness of a particular individual is a function of a vector of objective function values. Multiobjective problems are characterised by having no single unique solution, but a family of equally fit solutions with different values of decision variables. Care should therefore be taken to adopt some mechanism to ensure that the population is able to evolve the set of Pareto optimal solutions, for example by using fitness sharing [31] in the selection method. Although not supported in this version of the Genetic Algorithm Toolbox, it is planned that multiobjective search will be implemented in future versions.
Support for Multiple Populations

The GA Toolbox provides support for multiple subpopulations through the use of high-level genetic operator functions and a routine for exchanging individuals between subpopulations. In the literature, the use of multiple populations has been shown, in most cases, to improve the quality of the results obtained using GAs compared to the single population GA (see, for example, [32] and [33]).

The GA Toolbox supports the use of a single population divided into a number of subpopulations or *demes* by modifying the use of data structures such that subpopulations are stored in contiguous blocks within a single matrix. For example, the chromosome data structure, \( \text{Chrom} \), composed of \( \text{SUBPOP} \) subpopulations each of length \( N \) individuals is stored as:

\[
\begin{bmatrix}
\text{Ind}_1 \text{SubPop}_1 \\
\text{Ind}_2 \text{SubPop}_1 \\
\vdots \\
\text{Ind}_N \text{SubPop}_1 \\
\text{Ind}_1 \text{SubPop}_2 \\
\text{Ind}_2 \text{SubPop}_2 \\
\vdots \\
\text{Ind}_N \text{SubPop}_2 \\
\vdots \\
\text{Ind}_1 \text{SubPop}_{\text{SUBPOP}} \\
\text{Ind}_2 \text{SubPop}_{\text{SUBPOP}} \\
\vdots \\
\text{Ind}_N \text{SubPop}_{\text{SUBPOP}}
\end{bmatrix}
\]

This is known as the *Migration*, or *Island*, model [34]. Each subpopulation is evolved over generations by a traditional GA and from time to time individuals migrate from one subpopulation to another. The amount of migration of individuals and the pattern of that migration determines how much genetic diversity can occur.

To allow the Toolbox routines to operate independently on subpopulations, a number of high-level entry functions are provided that accept an optional argument that determines the number of subpopulations contained in a data structure. The low-level routines are then called independently, in turn, with each subpopulation...
to perform functions such as selection, crossover and reinsertion. These high-level functions are listed in the table below.

<table>
<thead>
<tr>
<th>SUBPOPULATION SUPPORT FUNCTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>mutate</td>
</tr>
<tr>
<td>recombin</td>
</tr>
<tr>
<td>reins</td>
</tr>
<tr>
<td>select</td>
</tr>
</tbody>
</table>

**Note:** As currently implemented, all subpopulations must be of equal size.

The transfer of individuals between subpopulations is implemented in the Toolbox function `migrate`. A single scalar is used to determine the amount of migration of individuals from one subpopulation to another. Thus, given a population comprised of a number of subpopulations, the same number of individuals will always be transferred from a subpopulation as the number it will receive from another subpopulation. A second parameter to the function `migrate` controls the manner in which individuals are selected for migration, either uniformly or according to fitness. Uniform selection picks individuals for migration and replaces individuals in a subpopulation with immigrants in a random manner. Fitness-based migration selects individuals according to their fitness level, the most fit individuals being the ones selected for migration, and replaces individuals in a subpopulation uniformly at random.

A further parameter specifies the population topology over which migration will take place. Fig. 7 shows the most basic migration paths implemented in `migrate`, the ring topology. Here individuals are transferred between directionally adjacent subpopulations. For example, individuals from subpopulation 6 migrate only to subpopulation 1 and individuals from subpopulation 1 only migrate to subpopulation 2.

![Figure 7: Ring Migration Topology](image-url)
A similar strategy to the ring topology is the neighbourhood migration of Fig. 8. Like the ring topology, migration is made only between nearest neighbours; however, migration may occur in either direction between subpopulations. For each subpopulation, the possible immigrants are determined, according to the desired selection method, from adjacent subpopulations and a final selection made from this pool of individuals. This ensures that individuals will not migrate from a subpopulation to the same subpopulation.

![Figure 8: Neighbourhood Migration Topology](image)

The most general migration strategy supported by `migrate` is that of unrestricted migration, Fig. 9. Here, individuals may migrate from any subpopulation to another. For each subpopulation, a pool of potential immigrants is constructed from the other subpopulations. The individual migrants are then determined according to the appropriate selection strategy.

![Figure 9: Unrestricted Migration Topology](image)

An example of a GA with multiple subpopulations is considered in the *Examples* Section.
Examples

This Section contains two detailed examples using the GA Toolbox to solve optimization problems:

- A simple binary GA to solve De Jong’s first test function [13].
- A real-valued multi-population GA to solve the Harvest problem [9].

The Simple GA

This example demonstrates how a simple GA can be constructed using routines from the GA Toolbox to solve an optimization problem. The objective function to be minimized is an extended version of De Jong’s first test function [13]:

\[ f_1(x) = \sum_{i=1}^{n} x_i^2, \quad -512 \leq x_i \leq 512 \]

where \( n \) defines the number of dimensions of the problem. For this example, we choose \( n = 20 \). The minimum of this function is, of course, located at \( x_i = 0 \).

The computational element of the MATLAB objective function is encapsulated in the code segment below.

```matlab
function ObjVal = objfun1( Phen )
    ObjVal = sum((Phen .* Phen)');
```

An m-file implementing this objective function, `objfun1`, is included with the GA Toolbox software.

Having written an m-file for the objective function, the GA code may now be constructed. This can be done directly from the MATLAB command line, in a script file or as a MATLAB function. Fig. 10 shows an outline of the script file `sga` supplied with the toolbox that implements a simple GA to solve this problem.

The first five lines describe the major variables of the GA. The number of individuals is set to \( \text{NIND} = 40 \) and the number of generations \( \text{MAXGEN} = 300 \). The number of variables used is \( \text{NVAR} = 20 \) and each variable uses a 20 bit representation, \( \text{PREC} = 20 \). This example uses a generation gap, \( \text{GGAP} = 0.9 \), and fitness-based reinsertion to implement an elitist strategy whereby the four most fit individuals always propagate through to successive generations. Thus, 36 (\( \text{NIND} \times \text{GGAP} \)) new individuals are produced at each generation.
NIND = 40; % Number of individuals  
MAXGEN = 300; % Maximum no. of generations  
NVAR = 20; % No. of variables  
PRECI = 20; % Precision of variables  
GGAP = 0.9; % Generation gap  

% Build field descriptor  
FieldD = [rep([PRECI],[1,NVAR]); ...  
            rep([-512;512],[1,NVAR]); rep([1;0;1;1],[1,NVAR])];  

% Initialise population  
Chrom = crtbp(NIND, NVAR*PRECI);  
gen = 0; % Counter  

% Evaluate initial population  
ObjV = objfun1(bs2rv(Chrom,FieldD));  

% Generational loop  
while gen < MAXGEN,  
  % Assign fitness values to entire population  
  FitnV = ranking(ObjV);  
  % Select individuals for breeding  
  SelCh = select('sus', Chrom, FitnV, GGAP);  
  % Recombine individuals (crossover)  
  SelCh = recombin('xovsp', SelCh, 0.7);  
  % Apply mutation  
  SelCh = mut(SelCh);  
  % Evaluate offspring, call objective function  
  ObjVSel = objfun1(bs2rv(SelCh,FieldD));  
  % Reinsert offspring into population  
  [Chrom ObjV] = reins(Chrom, SelCh, 1, ObjV, ObjVSel);  
  % Increment counter  
  gen = gen+1;  
end  

Figure 10: The Simple GA in MATLAB

The field descriptor is constructed using the matrix replication function, rep, to build the matrix, FieldD, describing the chromosomes’ representation and interpretation. In this case, FieldD describes 20 variables, each Gray coded using 20 bits over the interval [-512, 512]. An initial population is then created with the function crtbp thus,
Chrom = crtbp(NIND, NVAR*PRECI);

producing a matrix, Chrom, of NIND uniformly distributed random binary strings of length NVAR\times PRECI.

The generation counter, gen, is set to zero. The following line then converts the binary strings to real-values using the function bs2rv and evaluates the objective function, objfun1, for all of the individuals in the initial population as shown below.

\[ \text{ObjV} = \text{objfun1}(\text{bs2rv}(\text{Chrom}, \text{FieldD})) ; \]

The function bs2rv converts the binary strings in the matrix Chrom to real-values according to the field descriptor, FieldD, and returns a matrix of real-valued phenotypes. The return value matrix of bs2rv is then passed directly as the input argument to the objective function, objfun1, and the resulting objective function values are returned in the matrix ObjV. The GA then enters the generational while loop.

The first step in the generational loop is the assignment of fitness values to the individuals. In this example, rank-based fitness assignment is used as shown below.

\[ \text{FitnV} = \text{ranking}(\text{ObjV}) ; \]

Here, the objective function values, ObjV, are passed to the Toolbox function ranking with no other arguments. The default setting for the ranking algorithm assume a selective pressure of 2 and linear ranking, giving the most fit individual a fitness value of 2 and the least fit individual a fitness value of 0. Note that the ranking algorithm assumes that the objective function is to be minimised. The resulting fitness values are returned in the vector FitnV.

The selection stage uses the high-level function select to call the low-level stochastic universal sampling routine, sus, as follows,

\[ \text{SelCh} = \text{select}(\text{"sus"}, \text{Chrom}, \text{FitnV}, \text{GGAP}) ; \]

After selection, SelCh contains GGAP \times NIND individuals from the original population Chrom. These individuals are now recombined using the high-level function recombin as shown below.

\[ \text{SelCh} = \text{recombin}(\text{"xovsp"}, \text{SelCh}, 0.7) ; \]

recombin takes the individuals selected for reproduction, SelCh, and uses the single-point crossover routine, xovsp, to perform crossover with probability, Px = 0.7. The individuals in the input matrix SelCh are ordered such that individuals in odd numbered positions are crossed with the individual in the adjacent even numbered position. If the number of individuals in SelCh is odd then the last
individual is always returned uncrossed. The offspring produced by this crossover are returned in the same matrix SelCh. The actual crossover routine used may be changed by supplying a different function name in the string passed to `recombIn`.

Having produced a set of offspring, mutation may now be applied using the mutation function `mut`:

```c
SelCh = mut (SelCh);
```

Again, the offspring are returned in the matrix SelCh. As no probability of mutation has been specified in the function call, the default value of $Pm = 0.7/Lind = 0.0017$, where $Lind$ is the length of an individual, is assumed.

The objective function values for the offspring, ObjV Sel, may now be calculated thus:

```c
ObjV Sel = objfunl(bs2rv(SelCh, FieldD));
```

Because we have used a generation gap, the number of offspring is less than the size of the population. Therefore, we must reinsert the offspring into the current population. This is achieved using the reinsertion function `reins`, as follows:

```c
[Chrom, ObjV]=reins(Chrom, SelCh, 1, 1, ObjV, ObjV Sel);
```

Here, Chrom and SelCh are matrices containing the original population and the resulting offspring. The two occurrences of the numeral 1 indicate that a single population is used and that fitness-based reinsertion be applied. Fitness-based reinsertion replaces the least fit members of Chrom with the individuals in SelCh. The objective function values of the original population, ObjV, are thus required as a parameter to reins. In addition, so that the objective function values of the new population can be returned without having to re-evaluate the objective function for the entire population, the objective values of the offspring, ObjV Sel, are also supplied. reins returns the new population with the offspring inserted, Chrom, and the objective function values for this population, ObjV.

Finally, the generational counter, gen, is incremented. The GA iterates around the loop until gen = MAXGEN, in this case 300, and then terminates. The results of the genetic optimization are contained in the matrix ObjV and the values of the decision variables may be obtained by:

```c
Phen = bs2rv(Chrom, FieldD);
```
A Multi-population GA

This example shows how functions from the GA Toolbox may be used to implement a real-valued, multi-population GA. A MATLAB m-file script mpga, supplied with the Toolbox, implements the code described in this subsection. The objective function chosen is that of the harvest problem [9] which is a one-dimensional equation of growth:

$$x_{k+1} = a \cdot x_k - u_k,$$

with one equality constraint,

$$x_0 = x_N,$$

where $x_0$ is the initial condition of the state, $a$ is a scalar constant, and $x_k \in R$ and $u_k \in R^+$ are the state and nonnegative control respectively. The objective function is defined as:

$$J = \max \sum_{k=0}^{N-1} \sqrt{u_k},$$

where $N$ is the number of control steps over which the problem is to be solved. An m-file implementing this objective function, objharv, is supplied with the GA Toolbox software. Note that as this is a maximisation problem and the Toolbox routines are implemented to minimise, the objective function, objharv, multiplies $J$ by -1 to produce a minimisation problem. The initial condition is set to $x_0 = 100$ and the scalar is chosen as $a = 1.1$. Additionally, the exact optimal solution for this problem can be determined analytically as:

$$J^* = \sqrt{\frac{x_0 (a^N - 1)^2}{a^{N-1}(a-1)}}.$$

The number of control steps for this problem is $N = 20$, thus, NVAR = 20 decision variables will be used, one for each control input, $u_k$. The decision variables are bounded in the range RANGE = [0, 200], limiting the maximum control input, at any time-step, to 200. The field descriptor, FieldD, describing the decision variables may be constructed using the matrix replication function, rep, thus:

NVAR = 20;
RANGE = [0; 200];
FieldD = rep(RANGE, [1, NVAR]);
The parameters for the GA may be specified using MATLAB variables. For this example the following parameters are defined:

\[
\begin{align*}
\text{GGAP} &= 0.8; & \text{Generation gap} \\
\text{XOVR} &= 1; & \text{Crossover rate} \\
\text{MUTR} &= 1/\text{NVAR}; & \text{Mutation rate} \\
\text{MAXGEN} &= 1200; & \text{Maximum no. of generations} \\
\text{INSR} &= 0.9; & \text{Insertion rate} \\
\text{SUBPOP} &= 8; & \text{No. of subpopulations} \\
\text{MIGR} &= 0.2; & \text{Migration rate} \\
\text{MIGGEN} &= 20; & \text{No. of gens / migration} \\
\text{NIND} &= 20; & \text{No. of individuals / subpop}
\end{align*}
\]

As well as the conventional GA parameters, such as generation gap (GGAP) and crossover rate (XOVR), a number of other parameters associated with multi-population GAs are defined. Here, INSR = 0.9 specifies that only 90% of the individuals produced at each generation are reinserted into the population, SUBPOP = 8 subpopulations are to be used with a migration rate of MIGR = 0.2, or 20%, between subpopulations and migration occurring at every MIGGEN = 20 generations. Each subpopulation contains NIND = 20 individuals.

The functions used by the script-file are specified using MATLAB strings:

\[
\begin{align*}
\text{SEL}_F &= \text{’} \text{’sus’} \text{’}; & \text{Name of selection function} \\
\text{XOV}_F &= \text{’} \text{recdis’} \text{’}; & \text{Name of recombination function} \\
\text{MUT}_F &= \text{’} \text{mutbga’} \text{’}; & \text{Name of mutation function} \\
\text{OBJ}_F &= \text{’} \text{objharyv’} \text{’}; & \text{Name of objective function}
\end{align*}
\]

Because we are using discrete recombination, recdis, for the breeding of offspring, the crossover rate is not used and, hence XOVR = 1 above.

The initial population is created using the function crtrp and the generation counter, gen, set to zero:

\[
\begin{align*}
\text{Chrom} &= \text{crtrp(SUBPOP*NIND,FieldD)}; \\
\text{gen} &= 0;
\end{align*}
\]

This will consist of SUBPOP \times NIND individuals with individual decision variables chosen uniformly at random in the range specified by FieldD. The Chrom matrix contains all of the subpopulations and the objective function values for all the individuals in all the subpopulations may be calculated directly,

\[
\text{ObjV} = \text{feval(OBJ}_F, \text{Chrom)};
\]
using the MATLAB feval command. feval performs function evaluation taking the first input argument, in this case the name of our objective function, objharp, contained in OBJ_F, as the function to be evaluated and calls that function with all the remaining parameters as its input arguments. In this case, the function call is:

\[
\text{ObjV} = \text{objharp}(\text{Chrom});
\]

As a real-valued coding is used, there is no need to convert the chromosomes into a phenotypic representation. Like the previous example, the GA now enters a generational while loop.

The MATLAB code for the generational loop of the multi-population GA is shown in Fig. 11 below.

```
% Generational loop
while gen < MAXGEN,  
  % Fitness assignment to whole population
  FitnV = ranking(ObjV, 2, SUBPOP);  
  % Select individuals from population
  SelCh = select(SEL_F, Chrom, FitnV, GGAP, SUBPOP);  
  % Recombine selected individuals
  SelCh = recombin(XOV_F, SelCh, XOVR, SUBPOP);  
  % Mutate offspring
  SelCh = mutate(MUT_F, SelCh, FieldD, [MUTR], SUBPOP);  
  % Calculate objective function for offsprings
  ObjVOff = feval(OBJ_F, SelCh);  
  % Insert best offspring replacing worst parents
  [Chrom, ObjV] = reins(Chrom, SelCh, SUBPOP, ...  
                  [1 INSR], ObjV, ObjVOff);  
  % Increment counter
  gen = gen + 1;  
  % Migrate individuals between subpopulations
  if (rem(gen, MIGGEN) == 0)  
    [Chrom, ObjV] = ...  
        migrate(Chrom, SUBPOP, [MIGR, 1, 1], ObjV);  
  end
end
```

**Figure 11: Generational Loop of a Multipopulation GA**
The first step of the generational loop is the assignment of fitness values to individuals:

\[ \text{FitnV} = \text{ranking(ObjV, 2, SUBPOP)}; \]

Because we are using multiple subpopulations, ranking requires us to specify the selective pressure required, here we use a selective pressure of 2, and the number of subpopulations, SUBPOP. Each subpopulation’s individuals’ objective values in ObjV are ranked separately and the resulting sets of fitness values returned in the vector FitnV.

Within each subpopulation, individuals are selected for breeding independently using the high-level selection function, select:

\[ \text{SelCh} = \text{select(SEL_F, Chrom, FitnV, GGAP, SUBPOP)}; \]

select calls the low-level selection function. SEL_F = ‘sus’ for each subpopulation and builds the matrix SelCh containing all the pairs of individuals to be recombined. Like the previous example, the generation gap, GGAP = 0.8, means that 0.8 \times 20 = 16, GGAP \times NIND, individuals are selected from each subpopulation. Thus, SelCh contains a total of GGAP \times NIND \times SUBPOP = 128 individuals.

In a similar manner, the high-level recombination function, recombin, is used to recombine the pairs of individuals within each subpopulation of SelCh:

\[ \text{SelCh} = \text{recombin(XOV_F, SelCh, XOVR, SUBPOP)}; \]

The recombination function, XOV_F = ‘recdis’, performs discrete recombination between pairs of individuals for each subpopulation. As discrete recombination does not require the specification of a conventional crossover rate, the variable XOVR = 1.0 is used only for compatibility.

The offspring are now mutated:

\[ \text{SelCh} = \text{mutate(MUT_F, SelCh, FieldD, MUTR, SUBPOP)}; \]

Here, the breeder genetic algorithm mutation function, MUT_F = ‘mutbge’, is called using the high-level mutation routine, mutate, with a mutation rate of MUTR = 1/NIND = 0.05. The breeder genetic algorithm mutation function requires the field descriptor, FieldD, so that the result of mutation will not produce values outside the bounds of the decision variables.

The objective values of all the offspring, ObjVOff may now be calculated, again using feval:

\[ \text{ObjVOff} = \text{feval(OBJ_F, SelCh)}; \]
Offspring may now be reinserted into the appropriate subpopulations:

\[
[\text{Chrom}, \text{ObjV}] = \text{reins(Chrom, SelCh, SUBPOP, ...} \\
[1, \text{INSR}], \text{ObjV, ObjVOff});
\]

Fitness-based reinsertion is used, but the addition of the extra parameter to the fourth argument of `reins` specifies an insertion rate of `INSR = 0.9`. This means that for each subpopulation the least-fit 10% of the offspring are not reinserted.

Individuals in Multi-population GAs migrate between populations at some interval. The Toolbox routine `migrate` is used to swap individuals between subpopulations according to some migration strategy. In this example, at every `MIGGEN = 20` generations, migration takes place between subpopulations.

\[
\%
\text{Migration between subpopulations}
\]
\[
\text{if(rem(gen, MIGGEN) == 0)} \\
[\text{Chrom, ObjV}] = \text{migrate(Chrom, SUBPOP, ...} \\
[MIGR, 1, 1], \text{ObjV});
\]
\[
\text{end}
\]

Here, the most fit 20%, `MIGR = 0.2`, of each subpopulation is selected for migration. Nearest neighbour subpopulations then exchange these individuals amongst their subpopulations, uniformly reinserting the immigrant individuals (see the Support for Multiple Populations Section). The return matrix `Chrom` and vector `ObjV` reflect the changes of individuals in the subpopulations as a result of migration.

The GA iterates around the generational loop until `gen = MAXGEN` and then terminates. Fig. 12 shows a typical solution of the harvest problem obtained by `mpga`.

![Figure 12: Optimal Solution Obtained by mpga](image)

*Fig. 12: Optimal Solution Obtained by mpga*
Again, like the previous example, the results of the GA search are contained in the matrix \texttt{ObjV}. The objective value and index of the best individual are found using the function \texttt{min}, for example:

\[
[Y, I] = \text{min}(\texttt{ObjV})
\]

\[
Y = -73.2370
\]

\[
I = 50
\]

Remembering that the sign of the objective function has been changed to form a minimisation problem, these results correspond to an objective function value of 73.2370. The exact solution is given as 73.2376. The GA optimal solution is therefore accurate within a $10^{-5}$ error bound on the exact optimal solution. The chromosome values are displayed in Fig. 12 using:

\[
\text{plot(Chrom(I,:))}
\]
Demonstration Scripts

A number of test functions have been implemented for use with the GA script files supplied with the Toolbox. These test functions are supplied in a separate directory, test_fns, from the main demonstrations and Toolbox routines and are accompanied by a postscript file, test_fns.ps, giving full details of the problems implemented. The Table below summarises the test functions supplied with the Toolbox.

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2 Reference

This Section contains detailed descriptions of all of the functions in the Genetic Algorithm Toolbox. It begins with a list of functions grouped by subject area and continues with Reference entries in alphabetical order. Information about individual functions is also available through the on-line Help facility.
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</table>
Purpose
binary string to real value conversion

Synopsis
Phen = bs2rv(Chrom, FieldD)

Description
Phen = bs2rv(Chrom, FieldD) decodes the binary representation of the population, Chrom, into vectors of reals. The chromosomes are seen as concatenated binary strings of given length, and decoded into real numbers over a specified interval using either standard binary or Gray coding according to the decoding matrix, FieldD. The resulting matrix, Phen, contains the corresponding population phenotypes.

The use of Gray coding for binary chromosome representation is recommended as the regular Hamming distance between quantization intervals reportedly makes the genetic search less deceptive (see, for example, [1, 2]). An option to set the scaling between quantization points can be used to select either linear or logarithmic decoding to real values from binary strings. Logarithmic scaling is useful when the range of decision variable is unknown at the outset as a wider range of parametric values can be searched with fewer bits [3], thus reducing the memory and computational requirements of the GA.

The matrix FieldD has the following structure:

\[
\begin{bmatrix}
\text{len} \\
\text{lb} \\
\text{ub} \\
\text{code} \\
\text{scale} \\
\text{lbin} \\
\text{ubin}
\end{bmatrix}
\]

where the rows of the matrix are composed as follows:

len, a row vector containing the length of each substring in Chrom. Note that \text{sum(len)} should equal \text{length(Chrom)}.

lb and ub are row vectors containing the lower and upper bounds respectively for each variable used.

code is a binary row vector indicating how each substring is decoded. Select \text{code}(i) = 0 for standard binary and \text{code}(i) = 1 for Gray coding.
scale is a binary row vector indicating whether to use arithmetic and/or logarithmic scaling for each substring. Select scale(i) = 0 for arithmetic scaling and scale(i) = 1 for logarithmic scaling.

lbin and ubin are binary row vectors indicating whether or not to include each bound in the representation range. Select {l|u}bin(i) = 0 to exclude {l|u}b(i) from the representation range and {l|u}bin(i) = 1 to include {l|u}b(i) in the representation range.

Example

Consider the following binary population, created using the crtpb function, representing a set of single decision variables in the range [-1, 10]. The code extract shows how the function bs2rv may be used to convert the Gray code binary representation to real-valued phenotypes using arithmetic scaling.

Chrom = crtpb(4,8) % create random chromosomes

Chrom =
  0  0  0  0  0  1  1  1
  1  0  0  0  1  0  0  1
  0  0  1  0  1  0  0  0
  1  1  0  1  1  0  1  1

FieldD = [8; -1; 10; 1; 0; 1; 1]; % representation
Phen = bs2rv(Chrom,FieldD) % convert binary to real
Phen =
   -0.7843
    9.3961
    1.0706
    5.2980

Algorithm

bs2rv is implemented as an m-file in the GA Toolbox. If logarithmic scaling is used then the range must not include zero.

Reference


Purpose
Create a base vector.

Synopsis
BaseVec = crtbase(Lind, Base)

Description
crtbase produces a vector whose elements correspond to the base of the loci of
a chromosome structure. This function can be used in conjunction with crtbp
when creating populations using representations in different bases.

BaseVec = crtbase(Lind, Base) creates a vector of length Lind whose
individual elements are of base Base. If Lind is a vector, then
length(BaseVec) = sum(Lind). If Base is also a vector of
length(Lind), then BaseVec is composed of groups of bases of length
determined by the elements of Lind and base Base. This last option is useful
when describing populations with structure.

Example
To create a basevector for a population containing four elements in base 8 and five
elements in base four:
BaseV = crtbase([4 5], [8 4])
BaseV =
8 8 8 8 4 4 4 4 4

See Also
crtbp, bs2rv
crtbp

Purpose
Create an initial population.

Synopsis
[Chrom, Lind, BaseV] = crtbp(Nind, Lind)
[Chrom, Lind, BaseV] = crtbp(Nind, Lind, Base)
[Chrom, Lind, BaseV] = crtbp(Nind, BaseV)

Description
The first step in a genetic algorithm is to create an initial population consisting of random chromosomes. crtbp produces a matrix, Chrom, containing random values in its elements.

Chrom = crtbp(Nind, Lind) creates a random binary matrix of size \(Nind \times Lind\), where \(Nind\) specifies the number of individuals in the population and \(Lind\) the length of the individuals. Additionally, Chrom = crtbp([Nind, Lind]) can be used to specify the dimensions of the chromosome matrix.

Chrom = crtbp(Nind, Lind, Base) produces a chromosome matrix of base Base. If Base is a vector, then the value of the elements of Base specify the base of the loci of the chromosomes. In this case, the second right hand side argument may be omitted, Chrom = crtbp(Nind, BaseV).

[Chrom, Lind, BaseV] = crtbp(Nind, BaseV) also returns the length of the chromosome structure, Lind, and the base of the chromosome loci in the vector BaseV.

Example
To create a random population of 6 individuals of length 8 where the first four loci are base eight and the last five loci are base four:

BaseV = crtbase([4 5], [8 4])
Chrom = crtbp(6, BaseV)
or
Chrom = crtbp([6,8],[8 8 8 4 4 4 4])
Chrom =
    4 3 1 1 2 0 2 0 3
    1 4 7 5 2 1 1 1 0
    1 3 0 1 0 0 0 0 2
    1 5 5 7 2 0 2 3 1
    4 5 7 7 0 1 3 0 3
    4 2 4 0 3 3 1 1 0

Algorithm

crtbp is an m-file in the GA Toolbox that uses the MATLAB function rand.

See Also

crtbase, crtrp


**Purpose**
Create a real-valued initial population

**Synopsis**
Chrom = crtrp(Nind, FieldDR)

**Description**
The first step in a genetic algorithm is to create an initial population consisting of random individuals. *crtrp* produces a matrix, Chrom, containing uniformly distributed random values in its elements.

Chrom = crtrp(Nind, FieldDR) creates a random real-valued matrix of size Nind×Nvar, where Nind specifies the number of individuals in the population and Nvar the number of variables of each individual. Nvar is derived from FieldDR with Nvar = size(FieldDR, 2).

FieldDR (FieldDescriptionRealvalue) is a matrix of size 2×Nvar and contains the boundaries of each variable of an individual. The first row contains the lower bounds, the second row the upper bounds.

FieldDR is used in other functions (mutation).

**Example**
To create a random population of 6 individuals with 4 variables each:
Define boundaries on the variables.

FieldDR = [
    -100 -50 -30 -20; % lower bound
    100 50 30 20]; % upper bound

Create initial population
Chrom = crtrp(6, FieldDR)

Chrom =
    40.23  -17.17   28.95   15.38
    82.06   13.26  13.35  -9.09
    52.43  25.64  15.20  -2.54
   -47.50   49.10   9.09  10.65
   -90.50 -13.46 -25.63  -0.89
    47.21 -25.29   7.89 -10.48

**See Also**
mutbga, recdis, recint, reclin
migrate

Purpose
Migrate individuals between subpopulations

Synopsis
Chrom = migrate(Chrom, SUBPOP)
Chrom = migrate(Chrom, SUBPOP, MigOpt)
Chrom = migrate(Chrom, SUBPOP, MigOpt, ObjV)
[Chrom, ObjV] = migrate(Chrom, SUBPOP, MigOpt, ObjV)

Description
migrate performs migration of individuals between subpopulations in the current population, Chrom, and returns the population after migration, Chrom. Each row of Chrom corresponds to one individual. The number of subpopulations is indicated by SUBPOP. The subpopulations in Chrom are ordered according to the following scheme:

\[
\text{Chrom} = \begin{bmatrix}
\text{Ind}_1 \text{SubPop}_1 \\
\text{Ind}_2 \text{SubPop}_1 \\
\vdots \\
\text{Ind}_N \text{SubPop}_1 \\
\text{Ind}_1 \text{SubPop}_2 \\
\text{Ind}_2 \text{SubPop}_2 \\
\vdots \\
\text{Ind}_N \text{SubPop}_2 \\
\text{Ind}_1 \text{SubPop}_{SUBPOP} \\
\text{Ind}_2 \text{SubPop}_{SUBPOP} \\
\vdots \\
\text{Ind}_N \text{SubPop}_{SUBPOP}
\end{bmatrix}
\]

All subpopulations must have the same number of individuals.

MigOpt is an optional vector with a maximum of 3 parameters:
MigOpt(1): scalar containing the rate of migration of individuals between subpopulations in the range [0, 1]. If omitted or NaN, MigOpt(1) = 0.2 (20%) is assumed. If the migration rate is greater than 0 at least one individual per subpopulation will migrate.

MigOpt(2): scalar specifying the migration selection method
0 - uniform migration
1 - fitness-based migration
If omitted or NaN, MigOpt(2) = 0 is assumed.

MigOpt(3): scalar indicating the structure of the subpopulations for migration
0 - complete net structure
1 - neighbourhood structure
2 - ring structure
If omitted or NaN, MigOpt(3) = 0 is assumed.

If MigOpt is omitted or NaN, then the default values are assumed.

ObjV is an optional column vector with as many rows as Chrom and contains the corresponding objective values for all individuals in Chrom. For fitness-based selection of individuals (MigOpt(2) = 1) ObjV is necessary. If ObjV is an input and output parameter, the objective values are copied according to the migration of the individuals. This saves the recomputation of the objective values for the whole population.

**Example**

Chrom = migrate(Chrom, SUBPOP) chooses 20% of the individuals of one subpopulation and replaces these individuals with uniformly chosen individuals from all other subpopulations. This process is done for each subpopulation. (MigOpt = [0.2, 0, 0])

Chrom = migrate(Chrom, SUBPOP, [NaN 1 NaN], ObjV)) chooses 20% of the individuals of one subpopulation and replaces these individuals with a selection of the fittest individuals (smaller ObjV) from all other subpopulations. (net structure) This process is repeated for each subpopulation.

[Chrom,ObjV] = migrate(Chrom,SUBPOP,[0.3 1 2],ObjV]) chooses 30% of the individuals of one subpopulation and replaces these individuals with the fittest individuals (smaller ObjV) from an adjacent subpopulation in a unidirectional ring structure. This process is repeated for each subpopulation. The first subpopulation receives its new individuals from the last subpopulation (SUBPOP). ObjV is returned according to the migration of individuals.
The migration scheme employed:
subpop1-->subpop2-->subpop3-->...-->subpopSUBPOP-->subpop1

[Chrom, ObjV] = migrate(Chrom, SUBPOP, [NaN NaN 1], ObjV)
chooses 20% of the individuals of one subpopulation and replaces these
individuals with uniformly chosen individuals from both adjacent subpopulations
in an one dimensional neighborhood structure. This process is repeated for each
subpopulation. The first subpopulation receives its new individuals from the last
(SUBPOP) and second subpopulation the last subpopulation from the first and
SUBPOP-1 subpopulation. ObjV is returned according to the migration of
individuals.
The migration scheme employed:
subpopSUBPOP-->subpop1<--subpop2<--...<--subpopSUBPOP<--subpop1

See Also
select, recombin, mutate, reins

Reference

Genetic Algorithms”, In Parallel Problems Solving from Nature, Lecture Notes in


Distributed Evolutionary Search Processes for Function Optimization”, Parallel
**Purpose**
Discrete mutation operator

**Synopsis**
\[
\text{NewChrom} = \text{mut}(\text{OldChrom}, P_m, \text{BaseV})
\]

**Description**
\text{mut} takes the representation of the current population and mutates each element with a given probability. To allow for varying bases in the chromosome and structured populations, \text{mut} allows an additional argument \text{BaseV} that specifies the base of the individual elements of a chromosome.

\[
\text{NewChrom} = \text{mut}(\text{OldChrom}, P_m) \text{ takes the current population, OldChrom, with each row corresponding to an individuals, and mutates each element with probability } P_m. \text{ If the mutation probability, } P_m, \text{ is omitted, } P_m=0.7/Lind \text{ is assumed, where } Lind \text{ is the length of the chromosome structure. This value is selected as it implies that the probability of any one element of a chromosome being mutated is approximately 0.5 (see [1]). Without a third input argument, } \text{mut} \text{ assumes that the population is binary coded.}
\]

\[
\text{NewChrom} = (\text{OldChrom}, P_m, \text{BaseV}) \text{ uses a third argument to specify the base of the mutation of the individual elements of the chromosomes. In this case, } \text{length(BaseV)} = Lind, \text{ where } Lind \text{ is the length of the chromosome structure.}
\]

\text{mut} \text{ is a low-level mutation function normally called by } \text{mutate.}

**Example**
Consider a binary population \text{OldChrom} with 4 individuals each of length 8:

\[
\text{OldChrom} = [
0 0 0 0 0 1 1 1; \\
1 0 0 0 1 0 0 1; \\
0 0 1 0 1 0 0 0; \\
1 1 0 1 1 0 1 1]
\]

Mutate \text{OldChrom} with default probability:

\[
\text{NewChrom} = \text{mut}(\text{OldChrom})
\]

Thus, \text{NewChrom} can become:
NewChrom =
    0 0 1 0 0 1 1 1
    1 1 0 0 0 0 0 1
    0 0 0 0 1 0 0 0
    1 1 0 1 1 0 1 1

The complement of a binary string is obtained by applying mutation with probability 1.

```matlab
mut([1 0 1 0 1 1 1 0], 1)
ans =
    0 1 0 1 0 0 0 1
```

**See Also**

mutate, mutbga

**Reference**

mutate

**Purpose**
Mutation of individuals (high-level function).

**Synopsis**

\[
\begin{align*}
\text{NewChrom} &= \text{mutate}(\text{MUT\_F}, \text{OldChrom}, \text{FieldDR}) \\
\text{NewChrom} &= \text{mutate}(\text{MUT\_F}, \text{OldChrom}, \text{FieldDR}, \text{MutOpt}) \\
\text{NewChrom} &= \text{mutate}(\text{MUT\_F}, \text{OldChrom}, \text{FieldDR}, \text{MutOpt}, \text{SUBPOP})
\end{align*}
\]

**Description**

\text{mutate} performs mutation of individuals from a population, \text{OldChrom}, and returns the mutated individuals in a new population, \text{NewChrom}. Each row of \text{OldChrom} and \text{NewChrom} corresponds to one individual.

\text{MUT\_F} is a string that contains the name of the low-level mutation function, e.g. \text{mutbga} or \text{mut}.

\text{FieldDR} is a matrix of size \(2 \times \text{Nvar}\) and contains the bounds of each variable of an individual (real-valued variables) or a matrix of size \(1 \times \text{Nvar}\) and contains the base of each variable (discrete-valued variables). If \text{FieldDR} is omitted, empty or \text{NaN}, a binary representation of the variables is assumed.

\text{MutOpt} is an optional parameter containing the mutation rate, the probability of mutating a variable of an individual. If \text{MutOpt} is omitted, a default mutation rate is assumed. For real-value mutation \text{MutOpt} can contain a second parameter specifying a scalar for shrinking the mutation range (see \text{mutbga}).

\text{SUBPOP} is an optional parameter and determines the number of subpopulations in \text{OldChrom}. If \text{SUBPOP} is omitted or \text{NaN}, \text{SUBPOP} = 1 is assumed. All subpopulations in \text{OldChrom} must have the same size.

**Example**
For examples, see \text{mutbga} (real-value mutation) and \text{mut} (discrete-value mutation).

**Algorithm**

\text{mutate} checks the consistency of the input parameters and calls the low-level mutation function. If \text{mutate} is called with more than one subpopulation then the low-level mutation function is called separately for each subpopulation.

**See Also**
\text{mutbga}, \text{mut}, \text{recombin}, \text{select}
**Purpose**

Mutation of real-valued population (mutation operator of the breeder genetic algorithm).

**Synopsis**

\[
\text{NewChrom} = \text{mutbga}(\text{OldChrom, FieldDR})
\]

\[
\text{NewChrom} = \text{mutbga}(\text{OldChrom, FieldDR, MutOpt})
\]

**Description**

*mutbga* takes the real-valued population, *OldChrom*, mutates each variable with given probability and returns the population after mutation, *NewChrom*.

*NewChrom* = *mutbga*(*OldChrom*, *FieldDR*, *MutOpt*) takes the current population, stored in the matrix *OldChrom* and mutates each variable with probability *MutOpt*(1) by addition of small random values (size of the mutation step). The mutation step can be shrunk with *MutOpt*(2).

*FieldDR* is a matrix containing the boundaries of each variable of an individual (see *crtrp*).

*MutOpt* is an optional vector with a maximum of two parameters:

*MutOpt*(1):

- scalar containing the mutation rate in the range \([0, 1]\).
  - If omitted or NaN, *MutOpt*(1) = 1/\(\text{Nvar}\) is assumed, where \(\text{Nvar}\) is the number of variables per individual defined by \(\text{size}(*\text{FieldDR},2)\).
  - This value is selected as it implies that the number of variables per individual mutated is approximately 1.

*MutOpt*(2):

- scalar containing a value in the range \([0, 1]\) for shrinking the mutation range.
  - If omitted or NaN, *MutOpt*(2) = 1 is assumed (no shrinking).

*mutbga* is a low-level mutation function normally called by *mutate*.

**Example**

Consider the following population with three real-valued individuals:

\[
\text{OldChrom} = [
\begin{array}{cccc}
40.2381 & -17.1766 & 28.9530 & 15.3883; \\
82.0642 & 13.2639 & 13.3596 & -9.0916; \\
52.4396 & 25.6410 & 15.2014 & -2.5435
\end{array}
\]

The bounds are defined as:
FieldDR = [
    -100  -50  -30  -20;
    100   50   30   20]

To mutate OldChrom with mutation probability 1/4 and no shrinking of the mutation range:

NewChrom = mutbga(OldChrom, FieldDR, [1/4 1.0])

mutbga produces an internal mask table, MutMx, determining which variable to mutate and the sign for adding delta (see Algorithm), e.g.

MutMx = [
    0  0  0  1;
    0  0 -1  0;
    0  0 -1 -1]

An second internal table, delta, specifies the normalized mutation step size, e.g.

delta = [
    0.2500  0.2500  0.2500  0.2500;
    0.0001  0.0001  0.0001  0.0001;
    0.2505  0.2505  0.2505  0.2505]

Thus, after mutation NewChrom becomes:

NewChrom =
40.2381  -17.1766  28.9530  20.0000
82.0642   13.2638  13.3559  -9.0916
52.4396   25.6410  -7.6858  -7.5539

NewChrom - OldChrom shows the mutation steps

NewChrom - OldChrom =
0       0       0      4.6117
0       -0.0037  0      0
0      -7.5156  -5.0104

Algorithm

The mutation of a variable is computed as follows:

mutated variable = variable + MutMx × range × MutOpt(2) × delta

MutMx = ±1 with probability MutOpt(1), (+ or - with equal probability)
else 0

range = 0.5 × domain of variable (search interval defined by FieldDR).

delta = \sum_{i=0}^{m-1} \alpha_i 2^{-i}, \alpha_i = 1 \text{ with probability } 1/m, \text{ else } 0, \ m = 20.

With m = 20, the mutation operator is able to locate the optimum up to a precision of range × MutOpt(2) × 2^{-19}.
The mutation operator `mutbga` is able to generate most points in the hypercube defined by the variables of the individual and the range of the mutation. However, it tests more often near the variable, that is, the probability of small step sizes is greater than that of larger step sizes.

**See Also**

`mutate, recdis, recint, reclin`

**Reference**

Purpose
Rank-based fitness assignment

Synopsis

\[ \text{FitnV} = \text{ranking}(\text{ObjV}) \]
\[ \text{FitnV} = \text{ranking}(\text{ObjV}, \text{RFun}) \]
\[ \text{FitnV} = \text{ranking}(\text{ObjV}, \text{RFun}, \text{SUBPOP}) \]

Description

ranking ranks individuals according to their objective values, ObjV, and returns a column vector containing the corresponding individual fitness values, FitnV. This function ranks individuals for minimisation.

RFun is an optional vector with 1, 2 or length(ObjV) parameters:

If RFun is a scalar in \([1, 2]\), linear ranking is assumed and the scalar indicates the selective pressure.

If RFun is a vector with 2 parameters:

RFun(1):
- scalar indicating the selective pressure
  - for linear ranking RFun(1) must be in \([1, 2]\)
  - for non-linear ranking RFun(1) must be in \([1, \text{length(ObjV)} - 2]\)
  - If NaN, RFun(1) = 2 is assumed.

RFun(2):
- ranking method
  - 0 - linear ranking
  - 1 - non-linear ranking

If RFun is a vector of length(ObjV), it should contain the fitness values to be assigned to each rank.

If RFun is omitted or NaN, linear ranking and a selective pressure of 2 are assumed.

SUBPOP is an optional parameter and indicates the number of subpopulations in ObjV. If SUBPOP is omitted or NaN, SUBPOP = 1 is assumed. All subpopulations in ObjV must have the same size.

If ranking is called with more than one subpopulation then the ranking is performed separately for each subpopulation.
Example
Consider a population with 10 individuals. The current objective values are:

\[ \text{ObjV} = [1; 2; 3; 4; 5; 10; 9; 8; 7; 6] \]

Evaluate the fitness with linear ranking and selective pressure 2:

\[ \text{FitnV} = \text{ranking}(\text{ObjV}) \]

\[ \text{FitnV} = \]
\[ 2.00 \]
\[ 1.77 \]
\[ 1.55 \]
\[ 1.33 \]
\[ 1.11 \]
\[ 0 \]
\[ 0.22 \]
\[ 0.44 \]
\[ 0.66 \]
\[ 0.88 \]

Evaluate the fitness with non-linear ranking and selective pressure 2:

\[ \text{FitnV} = \text{ranking}(\text{ObjV}, [2 1]) \]

\[ \text{FitnV} = \]
\[ 2.00 \]
\[ 1.66 \]
\[ 1.38 \]
\[ 1.15 \]
\[ 0.95 \]
\[ 0.38 \]
\[ 0.45 \]
\[ 0.55 \]
\[ 0.66 \]
\[ 0.79 \]

Evaluate the fitness with the values in RFun:

\[ \text{RFun} = [3; 5; 7; 10; 14; 18; 25; 30; 40; 50] \]

\[ \text{FitnV} = \text{ranking}(\text{ObjV}, \text{RFun}) \]
FitnV =
50
40
30
25
18
3
5
7
10
14

Evaluate the fitness with non-linear ranking and selective pressure 2 for 2
subpopulations in ObjV:

FitnV = ranking(ObjV, [2 1], 2)

FitnV =
2.00
1.28
0.83
0.53
0.34
0.34
0.53
0.83
1.28
2.00

Algorithm

The algorithms for both linear and non-linear ranking first sorts the objective
function values into descending order. The least fit individual is placed in position
1 in the sorted list of objective values and the most fit individual position Nind
where Nind is the number of individuals in the population. A fitness value is then
assigned to each individual depending on its position, Pos, in the sorted
population.

For linear ranking individuals are assigned fitness values according to:

FitnV(Pos) = 2 - SP + 2 × (SP - 1) × (Pos - 1)/(Nind - 1), and

for non-linear ranking according to:

FitnV(Pos) = \( \frac{Nind \times X^{Pos-1}}{Nind} \sum_{i=1}^{Nind} X(i) \),

where X is computed as the root of the polynomial:
$$0 = (SP - 1) \times X^{N_{ind}-1} + SP \times X^{N_{ind}-2} + \ldots + SP \times X + SP.$$  

The vector $FitnV$ is then unsorted to reflect the order of the original input vector, $ObjV$. 

**See Also**

`select.rws.sus`

**Reference**

Purpose
Discrete recombination

Synopsis
NewChrom = recdis(OldChrom)

Description
recdis performs discrete recombination between pairs of individuals in the current population, OldChrom, and returns a new population after mating, NewChrom. Each row of OldChrom corresponds to one individual.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix OldChrom is odd then the last row is not mated and added at the end of NewChrom. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each individual and a selection function (e.g. select) to select individuals with a probability related to their fitness in the current population.

recdis is a low-level recombination function normally called by recomb.
NewChrom = [
    40.23 13.26 28.95 -9.09; % Mask(1,:) parent1&2
    82.06 13.26 28.95 15.38; % Mask(2,:) parent1&2
    -47.50 25.64 9.09 -2.54; % Mask(3,:) parent3&4
    52.43 25.64 9.09 10.65] % Mask(4,:) parent3&4

As the number of individuals in the parent population, OldChrom, was odd, the last individual is appended without recombination to NewChrom and the offspring returned to the users workspace, thus

NewChrom =
    40.23 13.26 28.95 -9.09
    82.06 13.26 28.95 15.38
    -47.50 25.64 9.09 -2.54
    52.43 25.64 9.09 10.65
    -90.50 -13.46 -25.63 -0.89

Algorithm

Discrete recombination exchanges variable values between the individuals. For each variable the parent who contributes its variable value to the offspring is chosen randomly with equal probability.

Discrete recombination can generate the corners of the hypercube defined by the parents.

See Also
recombin, recint, reclin, ranking, sus, rws

Reference
**Purpose**

Intermediate recombination

**Synopsis**

\[
\text{NewChrom} = \text{recint}(\text{OldChrom})
\]

**Description**

\text{recint} performs intermediate recombination between pairs of individuals in the current population, \text{OldChrom}, and returns a new population after mating, \text{NewChrom}. Each row of \text{OldChrom} corresponds to one individual.

\text{recint} is a function only applicable to populations of real-value variables (and not binary or integer).

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix \text{OldChrom} is odd then the last row is not mated and added at the end of \text{NewChrom}. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function \text{ranking} to assign a fitness level to each individual and a selection function (e.g. \text{select}) to select individuals with a probability related to their fitness in the current population.

\text{recint} is a low-level recombination function normally called by \text{recombin}.

**Example**

Consider the following population with three real-value individuals:

\[
\text{OldChrom} = [\begin{array}{ccccc}
40.23 & -17.17 & 28.95 & 15.38; & \% \text{parent1} \\
82.06 & 13.26 & 13.35 & -9.09; & \% \text{parent2} \\
52.43 & 25.64 & 15.20 & -2.54] & \% \text{parent3}
\end{array}] 
\]

To perform intermediate recombination:

\[
\text{NewChrom} = \text{recint}(\text{OldChrom})
\]

New values are produced by adding the scaled difference between the parent values to the first parent (see \text{Algorithm} subsection). An internal table of scaling factors, \text{Alpha}, is produced, e.g.

\[
\text{Alpha} = [\begin{array}{cccc}
-0.13 & 0.50 & 0.32 & 0.16; & \% \text{for offspring1} \\
1.12 & 0.54 & 0.44 & 1.16] & \% \text{for offspring2}
\end{array}]
\]

Thus, after recombination \text{NewChrom} would become:
NewChrom = [ \\
34.40  -1.92  23.86  11.33;  % Alpha(1,:) parent1&2 \\
87.11  -0.59  21.98  -13.04]  % Alpha(2,:) parent1&2

As the number of individuals in the parent population, OldChrom, was odd, the last individual is appended without recombination to NewChrom and the offspring returned to the users workspace, thus:

NewChrom = \\
34.40  -1.92  23.86  11.33 \\
87.11  -0.59  21.98  -13.04 \\
52.43  25.64  15.20  -2.54

Algorithm
Intermediate recombination combines parent values using the following rule:

offspring = parent1 + Alpha * (parent2 - parent1)

where Alpha is a scaling factor chosen uniformly at random in the interval [-0.25, 1.25]. recint produces a new Alpha for each pair of values to be combined.

Intermediate recombination can generate any point within a hypercube slightly larger than that defined by the parents.

Intermediate recombination is similar to line recombination reclin. Whereas recint uses a new Alpha factor for each pair of values combined together, reclin uses one Alpha factor for each pair of parents.

See Also
recombin, recdis, reclin, ranking, sus, rws

Reference
**Purpose**
Line recombination

**Synopsis**
\[ \text{NewChrom} = \text{reclin(OldChrom)} \]

**Description**
reclin performs line recombination between pairs of individuals in the current population, OldChrom, and returns a new population after mating, NewChrom. Each row of OldChrom corresponds to one individual.

reclin is a function only applicable to populations of real-value variables (not binary or integer).

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix OldChrom is odd then the last row is not mated and added at the end of NewChrom. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each individual and a selection function (e.g. select) to select individuals with a probability related to their fitness in the current population.

reclin is a low-level recombination function normally called by recombin.

**Example**
Consider the following population with three real-value individuals:

\[
\begin{array}{ccccc}
40.23 & -17.17 & 28.95 & 15.38; & \% \text{ parent1} \\
82.06 & 13.26 & 13.35 & -9.09; & \% \text{ parent2} \\
52.43 & 25.64 & 15.20 & -2.54; & \% \text{ parent3}
\end{array}
\]

To perform line recombination:

\[ \text{NewChrom} = \text{reclin(OldChrom)} \]

New values are produced by adding the scaled difference between the parent values to the first parent (see Algorithm). An internal table of scaling factors, Alpha, is produced, e.g.

\[
\begin{array}{cccc}
0.78; & \% \text{ for producing offspring1} \\
1.05] & \% \text{ for producing offspring2}
\end{array}
\]

Thus, after recombination NewChrom would become:
NewChrom = [ 
    72.97  6.64  16.74  -3.77; % Alpha(1) parent1&2
    84.25  14.85  12.54  -10.37] % Alpha(2) parent1&2

As the number of individuals in the parent population, OldChrom, was odd, the last individual is appended without recombination to NewChrom and the offspring returned to the users workspace, thus:

NewChrom =
    72.97  6.64  16.74  -3.77
    84.25  14.85  12.54  -10.37
    52.43  25.64  15.20  -2.54

Algorithm
Line recombination combines parent values using the following rule:

    offspring = parent1 + Alpha × (parent2 - parent1)

where Alpha is a scaling factor chosen uniformly at random in the interval [-0.25, 1.25]. reclin produces a new Alpha for each pair of parents to be combined.

Line recombination can generate any point on a slightly longer line than that defined by the parents.

Line recombination is similar to intermediate recombination recint. Whereas reclin uses one Alpha factor for each pair of parents combined together, recint uses a new Alpha factor for each pair of values.

See Also
recombin, recdis, recint, ranking, sus, rws

Reference
**Purpose**
Line recombination with mutation features

**Synopsis**

```
NewChrom = recmut(OldChrom, FieldDR)
NewChrom = recmut(OldChrom, FieldDR, MutOpt)
```

**Description**

`recmut` performs line recombination with mutation features between pairs of individuals in the current population, `OldChrom`, and returns a new population after mating, `NewChrom`. Each row of `OldChrom` corresponds to one individual.

`FieldDR` is a matrix containing the boundaries of each variable of an individual (see `crtrp`).

`MutOpt` is an optional vector with a maximum of 2 parameters:

- `MutOpt(1)`: scalar containing the recombination rate in the range `[0, 1]`. If omitted or NaN, `MutOpt(1) = 1` is assumed.
- `MutOpt(2)`: scalar containing a value in the range `[0, 1]` for shrinking the recombination range. If omitted or NaN, `MutOpt(2) = 1` is assumed (no shrinking).

`recmut` is a function only applicable to populations of real-value variables (and not binary or integer).

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix `OldChrom` is odd then the last row is not mated and added at the end of `NewChrom`. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function `ranking` to assign a fitness level to each individual and a selection function (see `usu` or `rws`) to select individuals with a probability related to their fitness in the current population.

`recmut` uses features of the mutation operator of the Breeder Genetic Algorithm (see `mutbga`). Therefore, the calling syntax of this recombination function is identical to this of the mutation function `mutbga`.

`recmut` is a low-level recombination function normally called by `mutate`. 
Example

Consider the following population with four real-valued individuals:

```
OldChrom = [
    40.2381 -17.1766  28.9530  15.3883; % parent1
    82.0642  13.2639  13.3596 -9.0916; % parent2
    52.4396  25.6410  15.2014 -2.5435; % parent3
   -47.5381  49.1963  9.0954  10.6521] % parent4
```

The boundaries are defined as:

```
FieldDR = [
    -100  -50  -30  -20;
    100   50   30   20]
```

To perform line recombination with mutation features:

```
NewChrom = recmut (OldChrom, FieldDR)
```

`recmut` produces an internal mask table, RecMx, determining which pairs of parents to recombine (here recombine all pairs) and the sign for adding the recombination step (see Algorithm). e.g.

```
RecMx = [
    1  -1  -1  -1; % for producing offspring1 & 2
    -1  -1  -1  -1] % for producing offspring3 & 4
```

Two further internal tables, delta and Diff, specify the normalized recombination step size, e.g.

```
delta = [
    0.1250  0.1250  0.1250  0.1250; % for offspring1 & 2
    0.0005  0.0005  0.0005  0.0005] % for offspring3 & 4
```

```
Diff = [
    1.3937  1.0143 -0.5196 -0.8157; % for offspring1 & 2
    -10.5712  2.4906 -0.6456  1.3952] % for offspring3 & 4
```

Thus, after recombination NewChrom becomes:

```
NewChrom =
    57.6637 -23.5177  30.0000  17.4281
    64.6386  19.6050  11.4106 -11.1314
    52.9719  25.5783  15.2112 -2.5576
   -48.0704  49.2590  9.0856  10.6662
```

Algorithm

The offsprings of a pair of two parents are computed as follows:

```
offspring1 = parent1 + RecMx×range×MutOpt(2)×delta×Diff
offspring2 = parent2 + RecMx×range×MutOpt(2)×delta×(-Diff)
```
RecMx = ±1 with probability MutOpt (1), (- with probability 0.9)
else 0

range = 0.5 × domain of variable (search interval defined by FieldDR).

\[ \text{delta} = \sum_{i=0}^{m-1} \alpha_i 2^{-i}, \alpha_i = 1 \text{ with probability } 1/m, \text{ else 0, } m = 20. \]

\[ \text{Diff} = \frac{\text{parent2} - \text{parent1}}{\| \text{parent1} - \text{parent2} \|} \]

The recombination operator recmut generates offspring in a direction defined by the parents (line recombination). It tests more often outside the area defined by the parents and in the direction of parent1. The point for the offspring is defined by features of the mutation operator. The probability of small step sizes is greater than that of bigger steps (see mutbga).

**See Also**
mutate, mutbga, reclin

**Reference**


recombin

**Purpose**
Recombination of individuals (high-level function).

**Synopsis**

```plaintext
NewChrom = recombin(REC_F, Chrom)
NewChrom = recombin(REC_F, Chrom, RecOpt)
NewChrom = recombin(REC_F, Chrom, RecOpt, SUBPOP)
```

**Description**

recombin performs recombination of individuals from a population, Chrom, and returns the recombined individuals in a new population, NewChrom. Each row of Chrom and NewChrom corresponds to one individual.

REC_F is a string that contains the name of the low-level recombination function, e.g. recdis or xovsp.

RecOpt is an optional parameter specifying the crossover rate. If RecOpt is omitted or NaN, a default value is assumed.

SUBPOP is an optional parameter and determines the number of subpopulations in Chrom. If SUBPOP is omitted or NaN, SUBPOP = 1 is assumed. All subpopulations in Chrom must have the same size.

**Example**

For examples see recdis, recint, reclin, xovsp, xovdp and xovmp.

**Algorithm**

recombin checks the consistency of the input parameters and calls the low-level recombination function. If recombin is called with more than one subpopulation then the low-level recombination function is called separately for each subpopulation.

**See Also**

recdis, recint, reclin, xovsp, xovdp, xovsh, mutate, select
Purpose
Reinsertion of offspring in the population.

Synopsis
Chrom = reins(Chrom, SelCh)
Chrom = reins(Chrom, SelCh, SUBPOP)
Chrom = reins(Chrom, SelCh, SUBPOP, InsOpt, ObjVCh)
[Chrom, ObjVCh] = reins(Chrom, SelCh, SUBPOP, InsOpt, ObjVCh, ObjVSel)

Description
reins performs insertion of offspring into the current population, replacing parents with offspring and returning the resulting population. The offspring are contained in the matrix SelCh and the parents in the matrix Chrom. Each row in Chrom and SelCh corresponds to one individual.

SUBFCP is an optional parameter and indicates the number of subpopulations in Chrom and SelCh. If SUBPOP is omitted or NaN, SUBPOP = 1 is assumed. All subpopulations in Chrom and SelCh each must have the same size.

InsOpt is an optional vector with a maximum of 2 parameters:
InsOpt(1):
scalar indicating the selection method for replacing parents with offspring:
0 - uniform selection, offspring replace parents uniformly at random
1 - fitness-based selection, offspring replace least fit parents
If omitted or NaN, InsOpt(1) = 0 is assumed

InsOpt(2):
scalar containing the rate of reinsertion of offspring per subpopulation as a fraction of subpopulation size in the range [0, 1].
If omitted or NaN, InsOpt(2) = 1.0 is assumed.
If INSR = 0 no insertion takes place.
If INSR is not 1.0 ObjVSel is needed for selecting the best offspring for insertion (truncation selection between offspring).

If InsOpt is omitted or NaN, then the default values are assumed.

ObjVCh is an optional column vector containing the objective values of the individuals in Chrom. ObjVCh is needed for fitness-based reinsertion.

ObjVSel is an optional column vector containing the objective values of the individuals in SelCh. ObjVSel is required if the number of offspring is greater
than the number of offspring to be reinserted into the population. In this case, offspring are selected for reinsertion according to their fitness.

If ObjVCh is output parameter, ObjVCh and ObjVSel are needed as input parameters. The objective values are then copied, according to the insertion of the offspring, saving the recomputation of the objective values for the whole population.

**Example**

Consider a population of 8 parents, Chrom, and a population of 6 offspring, SelCh:

Chrom = [1; 2; 3; 4; 5; 6; 7; 8]
SelCh = [11; 12; 13; 14; 15; 16]

Insert all offspring in the population:

Chrom = reins(Chrom, SelCh)

Thus, a new population Chrom is produced, e.g.:

Chrom =
  12
  11
  15
  16
  5
  13
  14
  8

Consider the following ObjVCh vector for the parent population Chrom and ObjVSel for the offspring, SelCh:

ObjVCh = [21; 22; 23; 24; 25; 26; 27; 28];
ObjVSel = [31; 32; 33; 34; 35; 36]

Insert all offspring fitness-based, i.e. replace least fit parents:

Chrom = reins(Chrom, SelCh, 1, 1, ObjVCh)

Chrom =
  1
  2
  16
  15
  14
  13
  12
  11
Insert 50% of the offspring fitness-based and copy the objective values according the insertion of offspring:

\[
[\text{Chrom, ObjVCh}] = \text{reins(Chrom, SelCh, 1, [1 0.5],... ObjVCh, ObjVSel)}
\]

\[
\text{Chrom} = \\
1 \\
2 \\
3 \\
4 \\
5 \\
13 \\
12 \\
11 \\
\]

\[
\text{ObjVCh} = \\
21 \\
22 \\
23 \\
24 \\
25 \\
33 \\
32 \\
31 \\
\]

Consider Chrom and SelCh consist of 2 subpopulations. Insert all offspring in the appropriate subpopulations:

\[
\text{Chrom} = \text{reins(Chrom, SelCh, 2)}
\]

\[
\text{Chrom} = \\
12 \\
2 \\
13 \\
11 \\
14 \\
6 \\
15 \\
16 \\
\]

**See Also**

select
**Purpose**
Matrix replication.

**Synopsis**
\[ \text{MatOut} = \text{rep} (\text{MatIn}, \text{REPN}) \]

**Description**
\( \text{rep} \) is a low-level replication function. Not normally used directly, \( \text{rep} \) is called by a number of functions in the GA-Toolbox.

\( \text{rep} \) performs replication of a matrix, \( \text{MatIn} \), specified by the numbers in \( \text{REPN} \) and returns the replicated matrix, \( \text{MatOut} \).

\( \text{REPN} \) contains the number of replications in every direction. \( \text{REPN}(1) \) specifies the number of vertical replications, \( \text{REPN}(2) \) the number of horizontal replications.

**Example**
Consider the following matrix \( \text{MatIn} \):
\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8
\end{bmatrix}
\]

To perform matrix replication:
\[
\begin{align*}
\text{MatOut} &= \text{rep} (\text{MatIn}, \ [1 \ 2]) \\
\text{MatOut} &= \\
1 & 2 & 3 & 4 & 1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 & 5 & 6 & 7 & 8
\end{align*}
\]

\[
\begin{align*}
\text{MatOut} &= \text{rep} (\text{MatIn}, \ [2 \ 1]) \\
\text{MatOut} &= \\
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8
\end{align*}
\]

\[
\begin{align*}
\text{MatOut} &= \text{rep} (\text{MatIn}, \ [2 \ 3]) \\
\text{MatOut} &= \\
1 & 2 & 3 & 4 & 1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 & 5 & 6 & 7 & 8
\end{align*}
\]
Purpose
Roulette wheel selection

Synopsis
NewChrIx = rws(FitnV, Nsel)

Description
rws probabilistically selects Nsel individuals for reproduction according to their fitness, FitnV, in the current population.

NewChrIx = rws(FitnV, Nsel) selects Nsel individuals from a population using roulette wheel selection. FitnV is a column vector containing a performance measure for each individual in the population. This can be achieved by using the function ranking or scaling to assign a fitness level to each individual. The return value, NewChrIx, is the index of the individuals selected for breeding, in the order that they were selected. The selected individuals can be recovered by evaluating Chrom(NewChrIx, :).

rws is a low-level selection function normally called by select.

Example
Consider a population of 8 individuals with the assigned fitness values, FitnV:

FitnV = [1.50; 1.35; 1.21; 1.07; 0.92; 0.78; 0.64; 0.5]

Select the indices of 6 individuals:

NewChrIx = rws(FitnV, 6)

Thus, NewChrIx can become:

NewChrIx =
2
5
1
1
3
7

Algorithm
A form of roulette wheel selection is implemented by obtaining a cumulative sum of the fitness vector, FitnV, and generating Nsel uniformly at random distributed numbers between 0 and sum(FitnV). The index of the individuals selected is determined by comparing the generated numbers with the cumulative sum vector. The probability of an individual being selected is then given by:
\[ F(x_i) = \frac{f(x_i)}{\sum_{i=1}^{N_{inds}} f(x_i)} \]

where \( f(x_i) \) is the fitness of individual \( x_i \) and \( F(x_i) \) is the probability of that individual being selected.

**See Also**
select, sus, reins, ranking, scaling

**Reference**

Purpose
Linear fitness scaling

Synopsis
 FitnV = scaling(ObjV, Smul)

Description
 scaling converts the objective values, ObjV, of a population into a fitness measure with a known upper bound, determined by the value of Smul, such that,

\[ F(x_i) = af(x_i) + b, \]

where \( f(x_i) \) is the objective value of individual \( x_i \), \( a \) is a scaling coefficient, \( b \) is an offset and \( F(x_i) \) is the resulting fitness value of individual \( x_i \). If \( f_{ave} \) is the average objective value in the current generation, then the maximum fitness of the scaled population is upper bounded at \( f_{ave} \times Smul \). If Smul is omitted then the default value of Smult = 2 is assumed. The average fitness of the scaled population is also set to \( f_{ave} \).

In the case of some of the objective values being negative, scaling attempts to provide an offset, \( b \), such that the scaled fitness values are greater than zero.

Algorithm
 scaling uses the linear scaling method described by Goldberg [1].

Note: linear scaling is not suitable for use with objective functions that return negative fitness values and is included here only for completeness.

See Also
 ranking, reins, rws, select, sus

Reference
**Purpose**
Selection of individuals from population (high-level function).

**Synopsis**
SelCh = select(SEL_F, Chrom, FitnV)
SelCh = select(SEL_F, Chrom, FitnV, GGAP)
SelCh = select(SEL_F, Chrom, FitnV, GGAP, SUBPOP)

**Description**
select performs selection of individuals from a population, Chrom, and returns
the selected individuals in a new population, SelCh. Each row of Chrom and
SelCh corresponds to one individual.

SEL_F is a string and contains the name of the low-level selection function, for
example rws or sus.

FitnV is a column vector containing the fitness values of the individuals in
Chrom. The fitness value indicates the expected probability of selection of each
individual.

GGAP is an optional parameter specifying the generation gap, the fraction of the
population to be reproduced. If GGAP is omitted or NaN, GGAP = 1.0 (100%) is
assumed. GGAP may also be greater than 1, allowing more offspring to be
produced than the number of parents. If Chrom consists of more than one
subpopulation, GGAP specifies the number of individuals to be selected per
subpopulation relative to the size of the subpopulation.

SUBPOP is an optional parameter and determines the number of subpopulations in
Chrom. If SUBPOP is omitted or NaN, SUBPOP = 1 is assumed. All
subpopulations in Chrom must have the same size.

**Example**
Consider a population of 8 individuals, Chrom, with the assigned fitness values,
FitnV:
Chrom = [
  1 11 21;
  2 12 22;
  3 13 23;
  4 14 24;
  5 15 25;
  6 16 26;
  7 17 27;
  8 18 28]

FitnV = [1.50; 1.35; 1.21; 1.07; 0.92; 0.78; 0.64; 0.5]

Select 8 individuals by stochastic universal sampling, sus:

SelCh = select('sus', Chrom, FitnV)

Thus, SelCh can become:

SelCh =
  7 17 27
  1 11 21
  6 16 26
  1 11 21
  5 15 25
  2 12 22
  3 13 23
  4 14 24

Consider Chrom consists of 2 subpopulations. Select 150% individuals per subpopulation by roulette wheel selection, rws:

FitnV = [1.50; 1.16; 0.83; 0.50; 1.50; 1.16; 0.83; 0.5]

SelCh = select('sus', Chrom, FitnV, 1.5, 2)

Thus, SelCh can become:

SelCh =
  3 13 23
  2 12 22
  1 11 21
  2 12 22
  2 12 22
  1 11 21
  6 16 26
  7 17 27
  7 17 27
  6 16 26
  7 17 27
  5 15 25
Algorithm

select checks the consistency of the input parameter and calls the low-level selection function. If select is called with more than one subpopulation then the low-level selection function is called separately for each subpopulation.

See Also

rws, sus, ranking, scaling, recombin, mutate
Purpose
Stochastic universal sampling

Synopsis
\[ \text{NewChrIx} = \text{sus(FitnV, Nsel)} \]

Description
\text{sus} probabilistically selects \text{Nsel} individuals for reproduction according to their fitness, \text{FitnV}, in the current population.

\[ \text{NewChrIx} = \text{rws(FitnV, Nsel)} \] selects \text{Nsel} individuals from a population using stochastic universal sampling [1]. \text{FitnV} is a column vector containing a performance measure for each individual in the population. This can be achieved by using the function \text{ranking} or \text{scaling} to assign a fitness level to each individual. The return value, \text{NewChrIx}, is the index of the individuals selected for breeding, in the order that they were selected. The selected individuals can be recovered by evaluating \text{Chrom(NewChrIx, :)}.

\text{sus} is a low-level selection function normally called by \text{select}.

Example
Consider a population of 8 individuals with the assigned fitness values, \text{FitnV}:

\[ \text{FitnV} = [1.50; 1.35; 1.21; 1.07; 0.92; 0.78; 0.64; 0.5] \]

Select the indices of 6 individuals:
\[ \text{NewChrIx} = \text{sus(FitnV, 6)} \]

Thus, \text{NewChrIx} can become:

\[ \text{NewChrIx} = \\
5 \\
6 \\
3 \\
1 \\
1 \\
2 \]

Algorithm
A form of stochastic universal sampling is implemented by obtaining a cumulative sum of the fitness vector, \text{FitnV}, and generating \text{Nsel} equally spaced numbers between 0 and \text{sum(FitnV)}. Thus, only one random number is generated, all the others used being equally spaced from that point. The index of the individuals
selected is determined by comparing the generated numbers with the cumulative sum vector. The probability of an individual being selected is then given by

\[ F(x_i) = \frac{f(x_i)}{\sum_{i=1}^{N_{ind}} f(x_i)}, \]

where \( f(x_i) \) is the fitness of individual \( x_i \) and \( F(x_i) \) is the probability of that individual being selected.

**See Also**
select, rws, reins, ranking, scaling

**Reference**
Purpose
Double-point crossover

Synopsis
NewChrom = xovdp(OldChrom, XOVr)

Description
xovdp performs double-point crossover between pairs of individuals contained in the current population, OldChrom, according to the crossover probability, XOVr, and returns a new population after mating, NewChrom. Each row of OldChrom and NewChrom corresponds to one individual. For the chromosomes any representation can be used.

XOVr is an optional parameter specifying the crossover rate. If XOVr is omitted, empty or NaN, XOVr = 0.7 is assumed.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix OldChrom is odd then the last row is not mated. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each chromosome and a selection function (select, sus or rws) to select individuals with a probability related to their fitness in the current population.

xovdp is a low-level crossover function normally called by recombin.

Algorithm
Consider the following two binary strings of the same length:

A1 = [1 1 0 1 0 1]
A2 = [1 0 1 0 1 0]

Double point crossover involves selecting uniformly at random two integer positions, k1 and k2, between 1 and length(A1), and swapping the variables in positions k1+1 to k2 between A1 and A2. Thus if the crossover positions k1 = 3 and k2 = 5, then A1 and A2 would become:

A1’ = [1 1 0 0 1 1]
A2’ = [1 0 1 1 0 0]

xovdp calls xovmp with the appropriate parameters.

See Also
xovdprs, xovsp, xovsh, xovmp, recombin, select
Purpose
Double-point reduced surrogate crossover

Synopsis
\[
\text{NewChrom} = \text{xovdprs}(\text{OldChrom}, \text{XOVR})
\]

Description
\text{xovdprs} performs double-point reduced surrogate crossover between pairs of individuals contained in the current population, \text{OldChrom}, according to the crossover probability, \text{XOVR}, and returns a new population after mating, \text{NewChrom}. Each row of \text{OldChrom} and \text{NewChrom} corresponds to one individual. For the chromosomes any representation can be used.

\text{XOVR} is an optional parameter specifying the crossover rate. If \text{XOVR} is omitted, empty or NaN, \text{XOVR} = 0.7 is assumed.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix \text{OldChrom} is odd then the last row is not mated. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function \text{ranking} to assign a fitness level to each chromosome and a selection function (\text{select}, \text{sus} or \text{rws}) to select individuals with a probability related to their fitness in the current population.

\text{xovdprs} is a low-level crossover function normally called by \text{recombin}.

Algorithm
For double point crossover see \text{xovdp}.

The \textit{reduced surrogate} operator constrains crossover to always produce new individuals wherever possible. This is implemented by restricting the location of crossover points such that crossover points only occur where gene values differ [1].

\text{xovdprs} calls \text{xovmp} with the appropriate parameters.

See Also
\text{xovdp, xovsprs, xovshrs, xovmp, recombin, select}

Reference
Purpose
Multi-point crossover

Synopsis
NewChrom = xovmp(OldChrom, XOVR, Npt, Rs)

Description
xovmp performs multi-point crossover between pairs of individuals contained in the current population, OldChrom, and returns a new population after mating, NewChrom. Each row of OldChrom and NewChrom corresponds to one individual. For the chromosomes any representation can be used.

XOVR is an optional parameter specifying the crossover rate. If XOVR is omitted, empty or NaN, XOVR = 0.7 is assumed.

Npt is an optional parameter specifying the number of crosspoints:
   0 - shuffle crossover.
   1 - single point crossover.
   2 - double point crossover.
   If Npt is omitted, empty or NaN, Npt = 0 is assumed.

Rs is an optional parameter specifying the use of reduced surrogate:
   0 - no reduced surrogate.
   1 - use reduced surrogate.
   If Rs is omitted, empty or NaN, Rs = 0 is assumed.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix OldChrom is odd then the last row is not mated. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each chromosome and a selection function (select, sus or rws) to select individuals with a probability related to their fitness in the current population.

xovmp is a low-level crossover function called by all other crossover functions. If called by recombine xovmp performs shuffle crossover without reduced surrogate identical to xovsh.

Algorithm
The algorithms used in single-point, double-point and shuffle crossover are described in the xovsp, xovdp and xovsh Reference entries respectively. The algorithms used in single-point, double-point and shuffle crossover with reduced surrogates are described in the xovsp.rs, xovdp.rs and xovsh.rs Reference entries respectively.

See Also
xovsp, xovdp, xovsh, xovsp.rs, xovdp.rs, xovsh.rs, recombine
**Purpose**
Shuffle crossover

**Synopsis**
NewChrom = xovsh(OldChrom, XOVR)

**Description**
xovsh performs shuffle crossover between pairs of individuals contained in the current population, OldChrom, according to the crossover probability, XOVR, and returns a new population after mating, NewChrom. Each row of OldChrom and NewChrom corresponds to one individual. For the chromosomes any representation can be used.

XOVR is an optional parameter specifying the crossover rate. If XOVR is omitted, empty or NaN, XOVR = 0.7 is assumed.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix OldChrom is odd then the last row is not mated. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each chromosome and a selection function (select, sus or rws) to select individuals with a probability related to their fitness in the current population.

xovsh is a low-level crossover function normally called by recombin.

**Algorithm**
Shuffle crossover is single-point crossover (see xovsp), but before the bits are exchanged, they are randomly shuffled in both parents. After recombination, the bits in the offspring are unshuffled. This removes positional bias as the bits are randomly reassigned each time crossover is performed [1].

xovsh calls xovmp with the appropriate parameters.

**See Also**

xovshrs, xovsp, xovdp, xovmp, recombin, select

**Reference**

Purpose
Shuffle crossover with reduced surrogate

Synopsis
NewChrom = xovshrs(OldChrom, XOVR)

Description
xovshrs performs shuffle crossover with reduced surrogates between pairs of individuals contained in the current population, OldChrom, according to the crossover probability, XOVR, and returns a new population after mating, NewChrom. Each row of OldChrom and NewChrom corresponds to one individual. For the chromosomes any representation can be used.

XOVR is an optional parameter specifying the crossover rate. If XOVR is omitted, empty or NaN, XOVR = 0.7 is assumed.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix OldChrom is odd then the last row is not mated. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each chromosome and a selection function (select, sus or rws) to select individuals with a probability related to their fitness in the current population.

xovshrs is a low-level crossover function normally called by recombin.

Algorithm
For shuffle crossover algorithm see xovsh.

The reduced surrogate operator constrains crossover to always produce new individuals wherever possible. This is implemented by restricting the location of crossover points such that crossover points only occur where gene values differ [1].

xovshrs calls xovmp with the appropriate parameters.

See Also
xovsh, xovspars, xovdprs, xovmp, recombin, select

Reference
**Purpose**
Single-point crossover

**Synopsis**
NewChrom = xovsp(OldChrom, XOVR)

**Description**

*xovsp* performs single-point crossover between pairs of individuals contained in the current population, *OldChrom*, according to the crossover probability, *XOVR*, and returns a new population after mating. *NewChrom*. *OldChrom* contains the chromosomes of the current population, each row corresponds to one individual. For the chromosomes any representation can be used.

*XOVR* is an optional parameter specifying the crossover rate. If *XOVR* is omitted, empty or NaN, *XOVR* = 0.7 is assumed.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix *OldChrom* is odd then the last row is not mated. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each chromosome and a selection function (select, sus or rws) to select individuals with a probability related to their fitness in the current population.

*xovsp* is a low-level crossover function normally called by *recomb*. 

**Algorithm**
Consider the following two binary strings of the same length:

A1 = [1 1 0 1 0 1]
A2 = [1 0 1 0 1 0]

Single-point crossover involves selecting uniformly at random an integer position, k, between 1 and (length(A1) - 1), and swapping the variables in positions k+1 to length(A1) between A1 and A2. Thus if the crossover position k = 3, then A1 and A2 would become:

A1′ = [1 1 0 0 1 0]
A2′ = [1 0 1 1 0 1]

*xovsp* calls *xovmp* with the appropriate parameters.

**See Also**
*xovsp*, *xovdp*, *xovsh*, *xovmp*, *recomb*, *select*
Purpose
Single-point reduced surrogate crossover

Synopsis
NewChrom = xovsprs(OldChrom, XOVR)

Description
xovsprs performs single-point reduced surrogate crossover between pairs of individuals contained in the current population, OldChrom, according to the crossover probability, XOVR, and returns a new population after mating. NewChrom. OldChrom contains the chromosomes of the current population, each row corresponds to one individual. For the chromosomes any representation can be used.

XOVR is an optional parameter specifying the crossover rate. If XOVR is omitted, empty or NaN, XOVR = 0.7 is assumed.

The pairs are mated in order, odd row with the next even row. If the number of rows in the matrix OldChrom is odd then the last row is not mated. The population should therefore be organised into contiguous pairs that require mating. This can be achieved by using the function ranking to assign a fitness level to each chromosome and a selection function (select, sus or rws) to select individuals with a probability related to their fitness in the current population.

xovsprs is a low-level crossover function normally called by recombin.

Algorithm
For single-point crossover see xovsp.

The reduced surrogate operator constrains crossover to always produce new individuals wherever possible. This is implemented by restricting the location of crossover points such that crossover points only occur where gene values differ [1].

xovsprs calls xovmp with the appropriate parameters.

See Also
xovsp, xovdp, xovdprs, xovsh, xovshrs, xovmp, recombin, select

Reference